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ADAPT: A Price-stabilizing Compliance Policy for Renewable Energy Certificates: The case of SREC markets

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Currently most Renewable Energy Certificate (REC) markets are defined based on targets which create an artificial step demand function resembling a cliff. This target policy produces volatile prices which make investing in renewables a risky investment. In this paper, we propose an alternative policy called Adjustable Dynamic Assignment of Penalties and Targets (ADAPT) which uses a sloped compliance penalty designed to stabilize REC prices. To capture market behavior, we model the market as a dynamic programming problem to understand how the market might balance the decision to use a REC now versus holding it for the future. Then, we present and prove some of the properties of this market, and finally we show that this mechanism reduces the volatility of REC prices which will stabilize the market and encourage long-term investments in renewables.

Key words: Compliance Policy, Renewable Energy Certificates, Stochastic Dynamic Programming, Price Volatility, Mechanism Design

1. Introduction

To promote the use of renewable sources of energy such as wind, solar, geothermal and biomass, Renewable Portfolio Standards (RPSs) have been implemented in a number of states in the US and other countries. These are sets of regulations requiring more generation from renewable energy sources usually by obligating Load Serving Entities (LSEs) to obtain a percentage of their generation from renewable sources. A Renewable Energy Certificate (REC) market is one tool used by many states to implement these policies. According to a set of regulations, certified renewable generators are credited by RECs for each unit of electricity generation. All LSEs are required to comply with their regulatory obligations by submitting enough RECs each Energy Year (EY), and otherwise they are charged with a penalty called the Alternative Compliance Payment (ACP) for each REC they are short. These market-based tools are expected to provide a more efficient, competitive and innovative environment for increasing renewable energy supply and decreasing the cost of generation in comparison with other regulatory tools such as feed-in tariffs (Burns and Kang (2012), Naroditsky (2013)).

This particular market design, however, as we discuss more extensively in the next section, can result in undesirably volatile prices. Although REC markets incentivize investing in renewables, excess volatility can affect the amount of the investment negatively. Price volatility is mostly attributed to the artificial vertical demand curve imposed by regulations. A number of papers (such as Felder and Loxley (2012), Berry (2002), Kildegaard (2008), Marchenko (2008)) discuss extensively the problems arising from a vertical demand curve (or a cliff policy) such as an uncompetitive market, volatile prices, higher cost of investment (due to higher risk), and difficult policy evaluation.

In this paper, we propose an Adjustable Dynamic Assignment of Penalties and Targets (ADAPT) policy by introducing both a sloped penalty function and an adaptive mechanism for requirements which can adjust to supply and demand changes. Under this sloped penalty proposal, the effective ACP is a function of the total submitted RECs in each energy year. While the requirement is not directly a function of the submitted or generated RECs, it can also be chosen to be a function of last year's surplus (or shortage). The slope of the penalty function and the sensitivity to last year's surplus are both tunable regulatory parameters. We show in this paper that this mechanism can be used by policy makers to dampen the volatility of market prices which will stabilize the market and encourage long-term investments in solar. We conduct our study of the ADAPT policy using the same generation model as that provided in Coulon et al. (2013). The price dynamics are the same, with the exception that we extend the model to handle multiple vintage years (i.e. year of production) simultaneously (Coulon et al. (2013) model each vintage year separately), and we extend the model to a longer horizon.

This paper makes the following contributions.

1. We propose a sloped target policy (which we call ADAPT), which can be used to encourage markets to achieve specific goals (e.g. energy from renewables, limiting emission,s ethanol targets, recyclable garbage, or water usage), without the instability that is experienced with more classical "cliff" policies which impose penalties when specific quantity targets are not met. The ADAPT policy is easily integrated into current markets, and represents a generalizable concept for hitting quantitative targets. Through a set of empirical and theoretical results, we show that the ADAPT policy produces prices that are dramatically more stable than would occur assuming optimal behavior with a cliff policy.
2. We derive an optimal policy for submitting RECs to the market, which describes the collective market behavior. We describe a series of structural results to accelerate the calculations, and then prove several properties of the optimal policy. These include the following:
   (a) We demonstrate how the optimal submission under ADAPT is chosen by market partici-pants to balance prices through time.
   (b) We show that the prices of RECs of different vintages (RECs generated in different years) are the same, which reduces the volatility of prices under the ADAPT policy.
   (c) We prove a property of the optimal solution that reduces the dimensionality across vintages to a scalar.
   (d) We prove that the total penalty payments under the sloped policy is bounded from below by the total payments of the cliff policy for a given submission level.

This paper is organized as follows. In section 2, we introduce and discuss the market design, regulations, and performance of the New Jersey SREC market, including the problems resulting from the current market design. In section 3, we introduce the ADAPT policy and describe how it can be implemented. In section 4, we formulate the collective behaviour of market participants using a stochastic dynamic programming model. In section 5, we characterize and prove some of the properties of the REC markets. In section 6, we calibrate our model to the NJ SREC market and discuss our methodology for solving this stochastic dynamic program. In section 7, we perform some experiments on different aspects of the ADAPT policy and its performance in comparison to the current cliff mechanisms. In section 8, we provide our concluding remarks.

2. Case Application: The New Jersey SREC Market

In the US, nearly 30 states have established RPSs and some of these use multipliers (e.g. 2 or 3 credits for each MWh of generation from solar) to specifically promote investment in solar energy. However, many LSEs prefer to use less expensive and more productive sources of renewable energy (in comparison with solar) such as wind energy to meet their REC obligations. Therefore, to even further promote the use of solar energy, 14 states have established separate solar set-asides and tradeable SRECs, and have been more successful in increasing investment in solar generation (Wiser et al. (2011), Witmer et al. (2012), Bird and Reger (2013), Bird et al. (2011), Glickstein (2013)).

The New Jersey (NJ) SREC market is the biggest in the US, and has the most ambitious target of over 4% generation from solar energy by 2028. The NJ SREC market has recorded the highest SREC prices of about $700 per SREC in year 2009, and NJ SREC generation has grown very rapidly from around 30,000 SRECs in EY2007 to more than 1,000,000 in EY2013. Each energy year represents the twelve month period ending on May 31 of that year. For example, EY2013 corresponds to the period between June 1 2012 and May 31 2013. Table 1 shows the requirement and Solar ACP (SACP) levels according to the latest rule change in 2012 (Bird and Reger (2013), DESIRE (2013), NJCEP (2013)).

The rules of the current SREC market in New Jersey can be summarized as follows.

- For each MWh of electricity generated by solar power plants, one SREC is issued to the owner of the power plant.

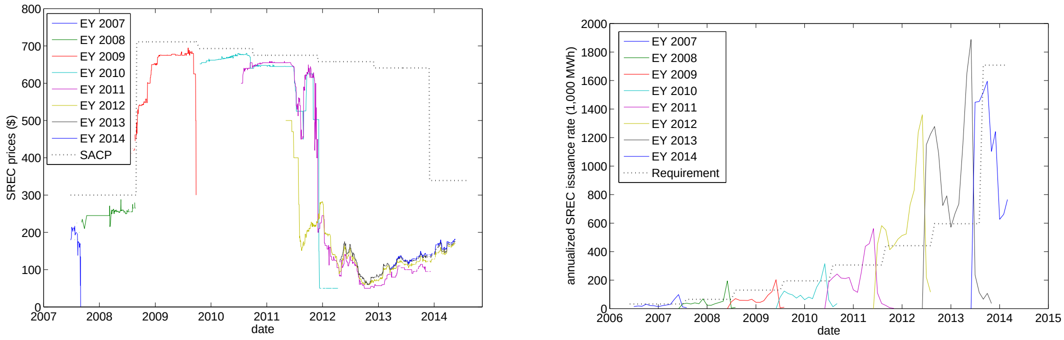
Table 1 The current requirement and penalty levels (last changed in 2012)

Energy Year	Target (% supply)	Target (projected MWh)	SACP
2012		442,000	$658.00
2013		596,000	$641.00
2014	2.05%	1,707,931	$339.00
2015	2.45%	2,071,803	$331.00
2016	2.75%	2,360,376	$323.00
2017	3.00%	2,613,580	$315.00
2018	3.20%	2,829,636	$308.00
2019	3.29%	2,952,857	$300.00
2020	3.38%	3,079,139	$293.00

- For several years into the future, the government sets targets for consumption of electricity from solar. These requirements are currently represented as a percentage of the total load.
- All LSEs, which are primarily utilities, are required to meet their requirement by submitting a sufficient number of SRECs each year. Otherwise, they need to pay a fixed penalty (Solar ACP, or SACP) for each megawatt-hour they fall below the target. They are free to generate SRECs themselves or to buy from other SREC generators.
- Finally, SRECs can be banked and used for a few years in the future. In the current New Jersey SREC market, SRECs can be banked and used for four years in addition to their production year.

Figure 1a shows historical SREC prices in NJ between 2007 and 2013. Shortly after its introduction, the NJ SREC market sustained very high prices, close to its SACP level, as the market struggled to meet the targets. However, several price drops and jumps can be observed in the historical prices. A market with such volatile prices is a risky environment for investors and thus decreases competition and increases the cost of generation (Felder and Loxley (2012)).

To reduce the volatility of SREC prices and thereby reduce the risk of solar investments, several rule changes have been introduced in the NJ SREC market. For example, higher SACP



(a) Daily average prices (b) Monthly SREC issuance rate

Figure 1 Historical NJ SREC prices and generation

levels and the possibility of banking (for two years) were introduced in 2008. In 2012, the banking horizon was extended to five years, along with increases in the targets. The frequency of changes in the market mechanism shows that these policy adjustments have not been a long-term solution and that market design has some room for improvement. To alleviate these problems, various alternatives are discussed by Felder and Loxley (2012), including price floors, long-term contracts and increased banking years. In addition they briefly suggest the use of a downward sloping demand curve to set non-zero SACP levels even when generation far exceeds the annual SREC requirement. However, as their exponential curve never reaches zero, there is no way to achieve compliance in such a scheme, removing somewhat the notion of a true quantity target. Furthermore, they provide only a rough qualitative sketch of the idea, leaving out the details of the implementation of their proposal, and making no attempt to model the resulting price dynamics.

3. ADAPT: A Price-stabilizing Compliance Policy

In this section, we introduce a new class of policies for computing alternative compliance penalties that allows regulators to avoid excess price volatility, which can discourage investment, and keep it within some tolerance levels. In this section, we provide a sketch of the policy, which we refer to as ADAPT, followed by a more rigorous mathematical model in section 4 which is used to understand how it behaves. We present the ADAPT policy in the framework of SREC markets.

The standard policy for assessing the alternative compliance payment used in New Jersey and elsewhere, looks like a cliff (see figure 2), where the SACP is assessed if usage is less than

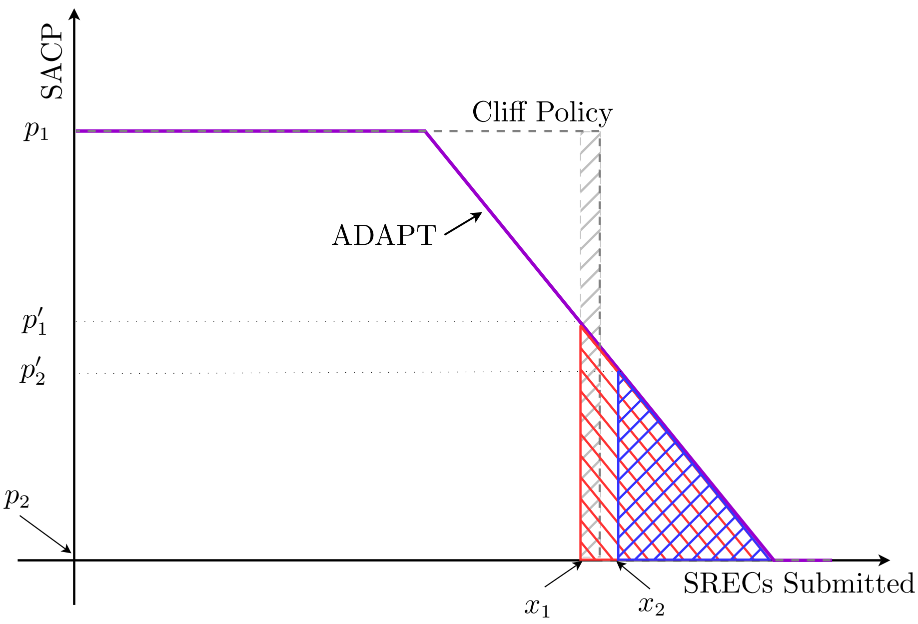


Figure 2 How a sloped SACP function can reduce price volatility (for two submission scenarios x_1 and x_2). The red and blue hashed areas represent total penalty payment under ADAPT for submission values x_1 and x_2 respectively.

the target dropping to zero if usage meets or exceeds the target. The distinguishing feature of ADAPT is to replace the cliff with a downward sloping function, as illustrated in figure 2. The idea of ADAPT is to reduce the large uncertainty in alternative compliance payments that has been observed to arise with a simple target-cliff style policy.

Figure 2 shows a step SACP function i.e. the current mechanism (the solid line), and its counterpart, a sloped SACP function (the purple dashed line). In this figure, x_1 and x_2 represent two scenarios of SREC submissions, with x_1 slightly smaller than the requirement and x_2 slightly larger than the requirement. Also, p_1 and p_2 are the penalty values of the current mechanism for x_1 and x_2 respectively, with p_1 an extremely high value while p_2 extremely low at zero. This means small changes in generation can result in high price volatility. On the other hand, with the sloped SACP function, the same submissions x_1 and x_2 result in penalties p'_1 and p'_2. These penalties are much closer than under the cliff policy, explaining why ADAPT produces more stable prices.

Also, the hashed gray area represents total penalty payment in the current mechanism for x_1, while total penalty payment for x_2 is zero. Note that the SREC requirement under the sloped mechanism (i.e. the right end of the sloped section) is more than the requirement of the current mechanism and so the penalty is paid for a higher number of SRECs (but a smaller penalty value for each unit). The hashed red and blue areas show the total penalty payment under the sloped mechanism for x_1 and x_2 respectively. The intuition behind this penalty payment (v.s. a rectangular payment in which the same SACP price p' is paid for all SRECs short of the requirement) is that in the slope region each SREC further from the right end of the sloped area should be penalized gradually more and more until ADAPT finally starts to penalize each one at the full SACP. This allows us to interpret the SACP function as a marginal demand curve, for which a total penalty equal to the area under the curve must be paid.

The sudden drop from a level close to the SACP to a level close to zero in the cliff policy happens specifically when transiting to a new year. For example, while the probability of the current level of generation not meeting this year's requirement may be high (i.e. SREC prices are high for this year), this probability may be low for the next year. This behavior can occur because targets are different every year, and generation may overtake a future target, producing much lower prices. The sloped SACP function can mitigate this problem by avoiding the binary nature of the cliff policy. An additional feature may also be incorporated to automatically redefine next year's requirement level, as we shall explain below.

The rules of the proposed ADAPT market can be summarized as follows.

- For each MWh of electricity generated by solar power plants, one SREC is issued to the owner of the power plant.

- The regulator sets SREC requirement levels for several years in the future. However, the requirement levels can be automatically adapted to generation levels according to the market performance in the last year. For example, in case of a surplus (shortage) in the last year, requirements will be increased (decreased) according to a formula.

- All Load Serving Entities (LSEs), which are primarily utilities, have to meet their requirement by submitting a sufficient number of SRECs each year. Otherwise, they need to pay a penalty for each megawatt-hour they fall below the target. Penalties, however, are calculated based on a sloped SACP function as represented in figure 2; the more SRECs submitted, the lower the penalty value (assuming being in the sloped area of the function).

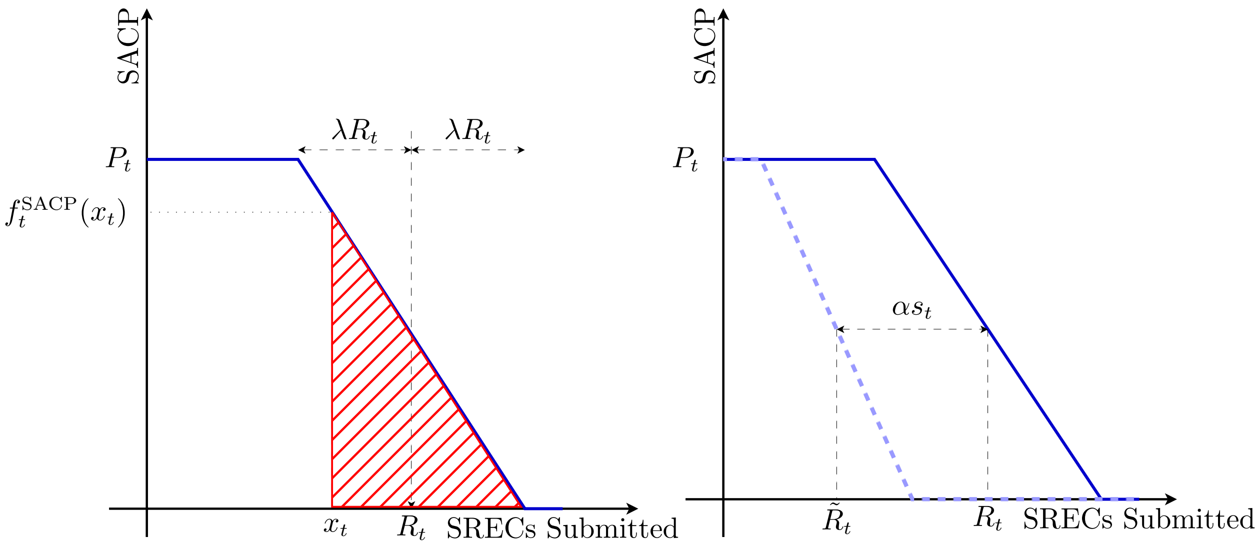


Figure 3 Sloped SACP function (left): the hashed area shows total market penalty payment and x_t represents the total submitted SRECs at the compliance time t ($t \in \mathbb{N}$). Adaptive requirements (right): how R_t is calculated from the fixed base requirement $\tilde{R}_t$ and surplus from the last compliance time (s_t).

- A generator (or LSE) holding SRECs can bank SRECs to the following year, raising the value of the SRECs that they are submitting now. This allows holders to more precisely balance the value of SRECs in the future against the price they receive now, but only within a specified range (due to the limited lifetime of each SREC).

Example 1. Assume that the sloped SACP function is defined as represented in figure 3 (the left plot) with $\lambda = 0.1$. Let $t = 14$ and 15 represent the compliance times of EY2014 and EY2015 respectively. According to the requirement and penalty figures given in table 1, the effective requirements of the sloped mechanism (the right end of the sloped area) for EY2014 will be equal to

$$(1 + \lambda)R_{14} = 1,878,724.$$

Also the maximum (but not the effective) penalty $P_{14}$and P_{15} is equal to the current SACP levels \$339 and \$331 respectively. Now assume that a total of 2,000,000 SRECs are available before the compliance time of EY2014. We call this value the number of banked SRECs at time $t = 14$, and represent it with $b_{14} = 2,000,000$. Assume that all market participants submit a collective of $x_t =$ 1,800,000 SRECs to the regulators, and the rest of SRECs are banked to be used in the future. According to the sloped SACP function, for this amount of submission, the SACP is equal to \$78. This means market participants are obliged to pay a total amount of $0.5 \times 78 \times 78,724$ dollars. SRECs are banked because their expected future price is higher than or equal to the penalty of \$78.

Here, total available SRECs at $t = 14$ has exceeded the target R_{14} (1,707,931, from table 1). Let s_t represent this surplus (or shortage if negative) from the last compliance time. In this case,

$$s_t = b_{14} - R_{14} = 292,069, \qquad t \in (14, 15].$$

In an adaptive requirement scheme, we increase the next year's requirement by a portion of s_t (as shown in the right plot of figure 3). For example, for $\alpha = 0.5$, we obtain:

$$\begin{aligned} R_{15} &= \tilde{R}_{15} + \alpha s_{15} \\ &= 2,071,803 + 0.5 \times 292,069 \\ &= 2,217,838, \end{aligned}$$

where, $\tilde{R}_{15}$ is equal to the already announced requirement for EY2015 (from table 1). We earlier assumed $s_{14} = 0$, and that is why we use $R_{14} = \tilde{R}_{14}$; however, we avoided asserting it before introducing the concept of surplus to prevent confusion.

Note that a (more complicated) formula similar to our adaptive requirement scheme is being used in the Massachusetts SREC market. We can use a mathematical model of this mechanism to investigate the effects of the sloped SACP function as well as other variations (and their combinations) to the current market. In addition to the sloped SACP function, these variations can include the possibility and the degree of adaptive requirements and/or number of banking years. Readers should be aware that the current mechanism is a special case of this general model, in which the slope factor and requirement adaptation factor are set to zero and SRECs are allowed to be banked for four years.

4. Mathematical Model

We consider a market with a sloped SACP function, adaptive requirements, and the possibility of banking for several years.

We use the following notation.

Indices (t, y): We index time by t, and energy year by y. In this model, the smallest time step is assumed to be a period of one month (i.e. $\Delta t = \frac{1}{12}$). Each energy year $y \in \mathbb{N}$ is associated with the time interval $[y-1, y]$, while $t = y$ determines the compliance time of energy year y. For example, time $t = 6$ corresponds to the end of energy year 2006 (May 31 2006). We also use y to index the vintage year of SRECs.

Parameters $(\tau, \lambda, \alpha, \tilde{R}_y, P_y, r)$:

- τ : The maximum number of years (compliance times) that an SREC can be banked.
- λ $(0 \leq \lambda \leq 1)$: The parameter determining the shape and slope of the SACP function (see figure 3). If $\lambda = 0$, we obtain the current cliff policy.
- α $(0 \leq \alpha \leq 1)$: The parameter representing the portion of last year's surplus (shortage) to be added to (deducted from) the base requirement.
- $\tilde{R}_y$: Number of SRECs required for energy year y, determined by the regulation and announced in advance. This, however, is not the effective requirement R_y, which is determined and updated according to market performance.
- P_y : The penalty set by the regulation to be paid for each SREC short of the requirement R_y in energy year y. We assume that P_y is decreasing in time, as these penalties have been decreasing sequences in all regulation changes in the NJ SREC market.
- r: Interest rate.

State variables: The state variable $S_t = ((b_{t,y})_y, \hat{g}_t, \bar{p}_t, s_t)$ is defined as follows.

- $(b_{t,y})_y$: Total number of accumulated or banked SRECs from different vintages at time t. Also, let b_t denote the total number of accumulated SRECs (valid for trading) at time t. If SRECs can be banked for τ years, at time t we have

$$b_t = \sum_{y=\max\{1, \lceil t \rceil - \tau\}}^{\lceil t \rceil} b_{t,y}.$$

- $\hat{g}_t$: The installed capacity of SREC generation at time t.
- $\bar{p}_t$: The (weighted and lagged) average historical price of SRECs up to time t. This is used to determine the rate of future solar installations.
- s_t : The surplus (or shortage) of SRECs from the last compliance time known at time t $(s_t = b_{\lceil t \rceil - 1} - \tilde{R}_{\lceil t \rceil - 1})$. This variable is needed for determining adaptive requirements.

Decision variables $((x_{t,y})_y)$*:*

- $(x_{t,y})_y$: The number of SRECs to be submitted to the market at time t from different vintage years. We represent the total number of SRECs submitted at time t by x_t defined as

$$x_t = \sum_y x_{t,y}.$$

Decisions will be made by a policy π using a function $X_t^\pi(S_t)$ to be determined later.

Exogenous information processes (W_t)*:* The exogenous information processes $W_t = (\varepsilon_t)$ is defined as the random variable indicative of noise in generation. Let $\omega \in \Omega$ be a sample path for $(W_1, \ldots, W_T)$. Let $\mathcal{F}_t = \sigma(W_1, \ldots, W_T)$ be the sigma-algebra on Ω, and let $\mathcal{P}$ be the probability measure on $(\Omega, \mathcal{F})$, giving us a probability space $(\Omega, \mathcal{F}, \mathcal{P})$. Throughout our presentation, we assume that any variable indexed by t is $\mathcal{F}_t$-measurable.

Other functions $(g_t(\bar{p}, \varepsilon_t), R_t(s_t),\ f_t^{SACP}(x_t))$*:*

- $g_t(\bar{p}, \varepsilon_t)$: Rate of SREC generation at time t. The generation rate g_t consists of an increasing trend of generation growth, seasonality, noise, and a feedback procedure to capture the dependence on the average historical prices of SRECs. We can represent g_t as:

$$g_t(\bar{p}, \varepsilon_t) = h_1(t - \lfloor t \rfloor) h_2(\varepsilon_t) \hat{g}_t(\bar{p}),$$

where functions h_1 and h_2 represent seasonality and random noise in the SREC rate of generation respectively. The increasing trend of installations and its dependance on the average historical prices of SRECs are captured by $\hat{g}_t(\bar{p})$.

According to our generation model, and supported by the historical generation and price data in New Jersey, we model g_t as

$$\hat{g}_t(\bar{p}) = \exp\left(a_0 + a_5 t + a_6 \sum_{u \in \{0, \Delta t, 2\Delta t, \ldots, t\}} \bar{p}_u \Delta t \right), \tag{1}$$

$$g_t(\bar{p}, \varepsilon_t) = \hat{g}_t(\bar{p}) \exp\Big(a_1 \sin(4\pi t) + a_2 \cos(4\pi t) + a_3 \sin(2\pi t) + a_4 \cos(2\pi t) + \varepsilon_t \Big). \tag{2}$$

- $R_t(s_t)$: The number of SRECs required by the regulation for time period t. This requirement is defined as

$$R_t(s_t) = \begin{cases} \tilde{R}_t + \alpha s_t & t \in \mathbb{N}, \\ 0 & t \notin \mathbb{N}. \end{cases}$$

- $f_t^{\text{SACP}}(x_t)$: The SACP function, determining the penalty price for any value of submission x_t at time t. This is the (artificial) inverse demand function of SRECs and at any compliance time ($t \in \mathbb{N}$) it can be represented by ($\lambda > 0$):

$$f_t^{\text{SACP}}(x_t) = \begin{cases} P_t, & x_t < (1-\lambda)R_t, \\ P_t - \frac{P_t}{2\lambda R_t}(x_t - (1-\lambda)R_t), & (1-\lambda)R_t \le x_t < (1+\lambda)R_t, \\ 0 & (1+\lambda)R_t \le x_t. \end{cases}$$

For other time periods $t \notin \mathbb{N}$, we define $f_t^{\text{SACP}}(x_t) = 0$. This definition helps us to present a more general and cleaner model without the need to discriminate between compliance and non-compliance times.

Transition functions: We represent the transition function generically using $S_{t+\Delta t} = S^M(S_t, (x_{t,y})_y,$
$W_{t+\Delta t})$.

According to equations (2) and (1),

$$\begin{aligned} \hat{g}_{t+\Delta t} &= \hat{g}_t \exp(a_5 \Delta t + a_6 \bar{p}_t \Delta t), \\ g_{t+\Delta t} &= \hat{g}_{t+\Delta t} \exp\Big(a_1 \sin(4\pi(t+\Delta t)) + a_2 \cos(4\pi(t+\Delta t)) \\ &\quad + a_3 \sin(2\pi(t+\Delta t)) + a_4 \cos(2\pi(t+\Delta t)) + \hat{\varepsilon}_{t+\Delta t}\Big). \end{aligned}$$

The number of banked SRECs $b_{t+\Delta t,y}$ can be obtained from

$$b_{t+\Delta t,y} = \begin{cases} 0 & y < \lceil t+\Delta t \rceil - \tau, \\ b_{t,y} - x_{t,y} & \lceil t+\Delta t \rceil - \tau \le y < \lceil t+\Delta t \rceil, \\ b_{t,y} + g_t \Delta t - x_{t,y} & y = \lceil t+\Delta t \rceil, \\ 0 & y > \lceil t+\Delta t \rceil. \end{cases}$$

This means that the expired SRECs are not banked any more, and of course, there is no SREC generation of future vintages. The newly generated SRECs are banked as current year SRECs, and total submissions in the last time step should also be taken into account. Note that based on the definition of the SACP and requirement functions, no SRECs would be submitted at non-compliance times ($x_{t,y} = 0$ if $t \notin \mathbb{N}$).

Also, the average historical price must be updated:

$$\bar{p}_{t+\Delta t} = \delta p_{t-\gamma, y^{max}} + (1-\delta)\bar{p}_t, \qquad 0 \le \delta \le 1$$

where $y^{max} = \lceil t \rceil$ represents the newest vintage year at time t, and γ represents the lag, i.e. the time between investment decision and actual generation.

Finally, surplus process can be updated as

$$s_{t+\Delta t} = \begin{cases} b_t - \tilde{R}_t & t \in \mathbb{N}, \\ s_t & t \notin \mathbb{N}. \end{cases}$$

Objective function: At each time step, the collective behaviour of a competitive market maximizes social welfare. Social welfare can be defined as the area under the (artificial) demand function minus the area under the marginal cost function. As the marginal cost of SREC generation is zero, social welfare is equal to the area under the SACP function. Let $C(S_t, x_t)$ denote the contribution function at time t, then we have

$$C(S_t, x_t) = \int_0^{x_t} f_t^{\text{SACP}}(u)du.$$

In order to obtain cleaner equations, we use $F_t(x_t)$ to represent $\int_0^{x_t} f_t^{\text{SACP}}(u)du$. Let Π be the set of all policies π i.e. functions that match each state S_t to a decision $X_t^\pi(S_t)$. We intend to find the best policy $\pi \in \Pi$ that maximizes $\mathbb{E}_t \sum_{t'=t}^{T} e^{-r(t'-t)} C(S_{t'}, X_{t'}^\pi(S_{t'}))$:

$$V_t(S_t) = \max_{\pi \in \Pi} \mathbb{E}_t \sum_{t'=t}^{T} e^{-r(t'-t)} F_{t'}(X_{t'}^\pi(S_{t'})),$$

where $e^{-r(t'-t)}$ is the discount factor, and $\mathbb{E}_t$ is a time t conditional risk-neutral expectation.

5. Model Properties

In this section, we derive a few properties of the model that help us in analysing the market and solving the dynamic programming model introduced in the previous section. We start by proving some features of the SREC markets that help us reduce the complexity of the stochastic dynamic program. Then, through a series of theorems and propositions, we prove different properties of the collective market behaviour. We use these results to obtain the optimal SREC submission policy representing the market behaviour for different market states. We then prove that (excluding very atypical circumstances) there is no price difference among different vintage years, which is one of the reasons why the ADAPT policy reduces price volatility. We also prove a theorem to compare total penalty payment under the two regimes.

Our first step in analysing this market is to reduce the number of decision variables to one variable. The following theorem shows that if we only solve the problem for the total number of banked SRECs, x_t, we can obtain the individual vintages, $x_{t,y}$, from x_t.

THEOREM 1. *If the optimal number of submitted SRECs at time t, x_t, is known, the optimal number of SRECs submitted from different vintages at time t is given by*

$$x_{t,y} = \begin{cases} 0 & y < \lceil t \rceil - \tau \\ \min\{b_{t,y}, x_t - \sum_{u=\lceil t \rceil - \tau}^{y-1} x_{t,u}\} & \lceil t \rceil - \tau \leq y \leq \lceil t \rceil \\ 0 & y > \lceil t \rceil \end{cases}$$

Proof. On one hand, all SREC vintages have the same contribution to the function $F_t(x_t)$ at each time (if they have not expired). On the other hand, the newer SRECs can be used further in the future to maximize $\mathbb{E}_t \sum_u e^{-r(u-t)} F_u(x_u)$. Therefore, for any y, if $x_{t,y} < b_{t,y}$, $x_{t,y+1} = 0$ (i.e. the older SRECs must be submitted first). This means that the oldest SRECs ($y = \lceil t \rceil - \tau$) must all be submitted ($x_{t,\lceil t \rceil - \tau} = b_{t,\lceil t \rceil - \tau}$) unless this value exceeds x_t. Therefore, we have $x_{t,\lceil t \rceil - \tau} = \min\{b_{t,\lceil t \rceil - \tau}, x_t\}$. Similarly, the second oldest SRECs must all be submitted if this does not exceed the total remaining SRECs from the optimal submission x_t, or $x_{t,\lceil t \rceil - \tau + 1} = \min\{b_{t,\lceil t \rceil - \tau + 1}, x_t - x_{t,\lceil t \rceil - \tau}\}$. Extending this to other vintages $\lceil t \rceil - \tau \leq y \leq \lceil t \rceil$, we obtain $x_{t,y} = \min\{b_{t,y}, x_t - \sum_{u=\lceil t \rceil - \tau}^{y-1} x_{t,u}\}$.

The expired SRECs ($y < \lceil t \rceil - \tau$) and SRECs that are not yet produced ($y > \lceil t \rceil$) of course cannot form a part of the optimal submitted SRECs x_t. □

SRECs are not a physical commodity and they do not have any storage or delivery costs or other constraints related to physical commodities. This means SREC prices must satisfy the martingale condition (at all times including compliance dates, as long as some SRECs are banked for the future). Thus, to ensure no arbitrage, at each time step, an SREC is valued at either the penalty price of that time period or the discounted expected SREC price in the next time period whichever is higher (under a risk-neutral measure). The expired SRECs do not have any value.

DEFINITION 1. Price of SRECs of vintage y at time t can be calculated as

$$p_{t,y} = \begin{cases} \max\{f_t^{\text{SACP}}(x_t), e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t,y}\}\}, & t \le y+\tau, \\ 0, & t > y+\tau. \end{cases} \tag{3}$$

Note that $f_t^{\text{SACP}}(x_t) = 0$ for times other than the compliance time ($t \notin \mathbb{N}$).

In order to obtain a proper policy for choosing x_t, we first need to prove the following theorem and propositions.

The following proposition shows how penalty value evolves through time under certain conditions.

PROPOSITION 1. *Let us represent the optimal policy at any compliance time $t \in \mathbb{N}$ by $x_t = X_t^*(S_t)$ ($\tau > 0$). Firstly, if $x_t < b_t$, then we have $f_t^{SACP}(x_t) \le\ e^{-r}\mathbb{E}_t\{f_{t+1}^{SACP}(x_{t+1})|x_t\}$. Also, if $b_{t,t-\tau} < x_t$, then we have $f_t^{SACP}(x_t) \ge\ e^{-r}\mathbb{E}_t\{f_{t+1}^{SACP}(x_{t+1})|x_t\}$.*

Proof. • $x_t < b_t \Rightarrow f_t^{\text{SACP}}(x_t) \le\ e^{-r}\mathbb{E}_t\{f_{t+1}^{\text{SACP}}(x_{t+1})|x_t\}$:

We prove this by contradiction. Let $x_t = X_t^*(S_t)$ represent the optimal compliance policy over time ($t \in \mathbb{N}$). Assume by contradiction that there exists t' for which we have $x_{t'} < b_{t'}$ and

$$f_{t'}^{\text{SACP}}(x_{t'}) > e^{-r}\mathbb{E}_{t'}\{f_{t'+1}^{\text{SACP}}(x_{t'+1})|x_{t'}\}.$$

We show that the policy $\hat{x} = \hat{X}_t(S_t)$, defined over all time periods as

$$\hat{X}_t(S_t) = \begin{cases} X_t^*(S_t) + \epsilon & t = t', \\ X_t^*(S_t) - \epsilon & t = t'+1, \\ X_t^*(S_t) & \text{Otherwise,} \end{cases}$$

gives a higher objective value than x_t for some small $\epsilon > 0$.

Note that $\hat{x}_t$ is a feasible solution, firstly because the total number of SRECs is still the same and submission has not exceeded the total available SRECs at each time (b_t). Also, $X_t^*(S_t)$ is always positive at the compliance times because P is decreasing in time, and, hence to maximize the objective function, it is always better that an available SREC (in the maximum flat area of $f_t^{\text{SACP}}(x_t)$) be submitted now rather than be banked for future. Note that $b_t > 0$, because generation rate is positive and thus there are some SRECs generated during the year.

Recall that $x_t = X_t^*(S_t)$ and $\hat{x}_t = \hat{X}_t(S_t)$. The objective function for $\hat{x}$ at any time t' can be obtained from

$$\mathbb{E}_{t'}\left\{\sum_{t=t'}^{T} F_t(\hat{x}_t)\right\} = \mathbb{E}_{t'}\left\{\sum_{t=t'}^{T} F_t(x_t)\right\} + \int_{x_{t'}}^{x_{t'}+\epsilon} f_{t'}^{\mathrm{SACP}}(u)du - e^{-r}\mathbb{E}_{t'}\int_{x_{t'+1}-\epsilon}^{x_{t'+1}} f_{t'+1}^{\mathrm{SACP}}(u)du.$$

As f_t^{SACP} and f_{t+1}^{SACP} are continuous decreasing functions, we have

$$\mathbb{E}_{t'}\left\{\sum_{t=t'}^{T} F_t(\hat{x}_t)\right\} > \mathbb{E}_{t'}\left\{\sum_{t=t'}^{T} F_t(x_t)\right\} + f_{t'}^{\mathrm{SACP}}(x_{t'}+\epsilon)\epsilon - e^{-r}\mathbb{E}_{t'}\{f_{t'+1}^{\mathrm{SACP}}(x_{t'+1}-\epsilon)\epsilon\}.$$

As from the initial assumption, $f_{t'}^{\mathrm{SACP}}(x_{t'}) > e^{-r}\mathbb{E}_{t'}\{f_{t'+1}^{\mathrm{SACP}}(x_{t'+1})|x_{t'}\}$, and f is a continuous function, we can always find $\epsilon > 0$ for which $f_{t'}^{\mathrm{SACP}}(x_{t'}+\epsilon) - e^{-r}\mathbb{E}_{t'}\{f_{t'+1}^{\mathrm{SACP}}(x_{t'+1}-\epsilon) > 0$. This means $\mathbb{E}_{t'}\left\{\sum_{t=t'}^{T} F_t(\hat{x}_t)\right\} > \mathbb{E}_{t'}\left\{\sum_{t=t'}^{T} F_t(x_t)\right\}$, which leads to a contradiction with x_t being the optimal decision and so proves that $f_t^{\mathrm{SACP}}(x_t) \leq e^{-r}\mathbb{E}_t\{f_{t+1}^{\mathrm{SACP}}(x_{t+1})|x_t\}$.

- $b_{t,t-\tau} < x_t \Rightarrow f_t^{\mathrm{SACP}}(x_t) \geq e^{-r}\mathbb{E}_t\{f_{t+1}^{\mathrm{SACP}}(x_{t+1})|x_t\}$:

We can use a similar methodology to prove this part of the proposition.

□

The following proposition provides the conditions under which SRECs are banked for future use.

Proposition 2. *Let $x_t = X_t^*(S_t)$ represent the optimal policy at all compliance times $t \in \mathbb{N}$. If $f_t^{SACP}(b_t) < e^{-r}\mathbb{E}_t\{f_{t+1}^{SACP}(x_{t+1})\}$, and $b_{t,t-\tau} < b_t$ then $x_t < b_t$.*

Proof. Let $x_t = X_t^*(S_t)$ be the optimal policy, and assume that there is some t' for which $X_{t'}^*(S_{t'}) = b_{t'}$ while $f_{t'}^{\mathrm{SACP}}(b_{t'}) < e^{-r}\mathbb{E}_{t'} f_{t'+1}^{\mathrm{SACP}}(X_{t'+1}^*(S_{t'+1}))$. We can show that $\hat{x}_t = \hat{X}_t(S_t)$ defined as follows provides a better objective function.

$$\hat{X}_t(S_t) = \begin{cases} X_t^*(S_t) - \epsilon & t = t', \\ X_t^*(S_t) + \epsilon & t = t'+1, \\ X_t^*(S_t) & \text{Otherwise,} \end{cases}$$

Firstly notice that $\hat{X}_t(S_t)$ provides a feasible solution, because $\hat{X}_t(S_t) = X_t^*(S_t)$, except that ϵ non-expiring SRECs have been submitted at time $t'+1$ instead of t'. Recall that $x_t = X_t^*(S_t)$ and $\hat{x}_t = \hat{X}_t(S_t)$. The objective function for $\hat{x}$ at time t' can be obtained from

$$\begin{aligned}\mathbb{E}_{t'}\left\{\sum_{t=t'}^{T} F_t(\hat{x}_t)\right\} &= \mathbb{E}_{t'}\left\{\sum_{t=t'}^{T} F_t(x_t)\right\} - \int_{b_{t'}-\epsilon}^{b_{t'}} f_{t'}^{\mathrm{SACP}}(u)du + e^{-r}\mathbb{E}_{t'}\int_{x_{t'+1}}^{x_{t'+1}+\epsilon} f_{t'+1}^{\mathrm{SACP}}(u)du \\ &\geq \mathbb{E}_{t'}\left\{\sum_{t=t'}^{T} F_t(x_t)\right\} - f_{t'}^{\mathrm{SACP}}(b_{t'}-\epsilon)\epsilon + e^{-r}\mathbb{E}_{t'}\{f_{t'+1}^{\mathrm{SACP}}(x_{t'+1}+\epsilon)\}\epsilon. \qquad (4)\end{aligned}$$

As $f_{t'}^{\text{SACP}}(b_{t'}) < e^{-r}\mathbb{E}_{t'}\{f_{t'+1}^{\text{SACP}}(x_{t'+1})\}$ and f functions are continuous, we can always find small enough $\epsilon > 0$ for which we have $f_{t'}^{\text{SACP}}(b_{t'} - \epsilon) < e^{-r}\mathbb{E}_{t'}\{f_{t'+1}^{\text{SACP}}(x_{t'+1} + \epsilon)\}$. Therefore, we have

$$\mathbb{E}_{t'}\left\{\sum_{t=t'}^{T} F_t(\hat{x}_t)\right\} > \mathbb{E}_{t'}\left\{\sum_{t=t'}^{T} F_t(x_t)\right\},$$

which contradicts the optimality of x_t. $\square$

The next proposition establishes conditions under which more than zero bankable SRECs (i.e. SRECs that are not expiring) are submitted.

Proposition 3. *Let $x_t = X_t^*(S_t)$ represent the optimal policy at all compliance times $t \in \mathbb{N}$. If $e^{-r}\mathbb{E}_t\{f_{t+1}^{SACP}(x_{t+1})\} < f_t^{SACP}(b_{t,t-\tau})$, and $b_{t,t-\tau} < b_t$, then $x_t > b_{t,t-\tau}$.*

Proof. Firstly, all SRECs that are about to expire ($b_{t,t-\tau}$) must be submitted at time $t \in \mathbb{N}$, because these SRECs can contribute to the objective function at time t but will not be available in the future. This means $x_t \geq b_{t,t-\tau}$. We show that under the proposition conditions, $x_t \neq b_{t,t-\tau}$.

Similar to the proof of proposition 2, we can show that there exists a small enough $\epsilon > 0$, for which $\hat{x}_t = \hat{X}_t(S_t)$ defined as follows is feasible and yields a better objective value than $(x_t)_t$ with $x_{t'} = b_{t',t'-\tau}$.

$$\hat{x}_t = \begin{cases} b_{t,t-\tau} + \epsilon & t = t', \\ x_t - \epsilon & t = t' + 1, \\ x_t & \text{Otherwise.} \end{cases}$$

$\square$

We can now prove the following theorem which we can use to describe the optimal policy.

Theorem 2. *If $\tau > 0$, then at any time $t \in \mathbb{N}$, the optimal policy $x_t = X_t^*(.)$ must solve*

$$f_t^{SACP}(x_t) = \min\{f_t^{SACP}(b_{t,t-\tau}), \max\{f_t^{SACP}(b_t), e^{-r}\mathbb{E}_t\{f_{t+1}^{SACP}(x_{t+1})|x_t\}\}\}.$$

Proof. As with the previous proof, we again have that all SRECs that are about to expire ($b_{t,t-\tau}$) must be submitted at time $t \in \mathbb{N}$, because these SRECs can contribute to the objective function at time t but will not be available in the future. Therefore, $x_t \geq b_{t,t-\tau}$ or $f_t^{\text{SACP}}(b_{t,t-\tau})$ is an upper bound for $f_t^{\text{SACP}}(x_t)$:

$$x_t \geq b_{t,t-\tau} \quad \text{or} \quad f_t^{\text{SACP}}(x_t) \leq f_t^{\text{SACP}}(b_{t,t-\tau}). \tag{5}$$

From (5), if $b_{t,t-\tau} = b_t$, then all SRECs must be submitted ($x_t = b_t$) which is consistent with the formula given by the theorem. So, from here onwards, we prove the theorem only for the case of $b_{t,t-\tau} < b_t$.

Case 1 $f_t^{\mathbf{SACP}}(b_t) > \; e^{-r}\mathbb{E}_t\{f_{t+1}^{\mathbf{SACP}}(x_{t+1})\} \Rightarrow x_t = b_t \; (f_t^{\mathbf{SACP}}(x_t) = f_t^{\mathbf{SACP}}(b_t))$ **:** We prove this by contradiction. Assume on the contrary that for the optimal x and a time t', we have $x_{t'} < b_{t'}$ while $f_{t'}^{\text{SACP}}(b_{t'}) > e^{-r}\mathbb{E}_{t'}\{f_{t'+1}^{\text{SACP}}(x_{t'+1})\}$. From proposition 1 we can conclude that as $x_{t'} < b_{t'}$, $f_{t'}^{\text{SACP}}(x_{t'}) \leq e^{-r}\mathbb{E}_{t'}\{f_{t'+1}^{\text{SACP}}(x_{t'+1})\}$. Also $f_{t'}^{\text{SACP}}$ is a non-increasing function and $x_{t'} < b_{t'}$, thus $f_{t'}^{\text{SACP}}(x_{t'}) \geq f_{t'}^{\text{SACP}}(b_{t'})$. However, this contradicts the condition $f_{t'}^{\text{SACP}}(b_{t'}) > e^{-r}\mathbb{E}_{t'}\{f_{t'+1}^{\text{SACP}}(x_{t'+1})\} \geq \; f_{t'}^{\text{SACP}}(x_{t'})$.

Case 2 $f_t^{\mathbf{SACP}}(b_t) \leq \; e^{-r}\mathbb{E}_t\{f_{t+1}^{\mathbf{SACP}}(x_{t+1})\} \Rightarrow f_t^{\mathbf{SACP}}(x_t) = \min\{f_t^{\mathbf{SACP}}(b_{t,t-\tau}),$ $e^{-r}\mathbb{E}_t\{f_{t+1}^{\mathbf{SACP}}(x_{t+1})\}\}$**:** We divide this case into subcases 2a and 2b.

Case 2a $f_t^{\text{SACP}}(b_t) < f_t^{\text{SACP}}(b_{t,t-\tau}) \leq e^{-r}\mathbb{E}_t\{f_{t+1}^{\text{SACP}}(x_{t+1})\} \Rightarrow x_t = b_{t,t-\tau} \; (f_t^{\text{SACP}}(x_t) = f_t^{\text{SACP}}(b_{t,t-\tau}))$: Proposition 2 ensures that $x_t \neq b_t$. Also, x_t cannot satisfy $b_{t,t-\tau} < x_t < b_t$. If this were true, according to proposition 1, $f_t^{\text{SACP}}(x_t)$ must be equal to $e^{-r}\mathbb{E}_t\{f_{t+1}^{\text{SACP}}(x_{t+1})\}$. There are two possibilities in this case. First, $b_{t,t-\tau}$ is in the sloped area of the SACP function, and so $b_{t,t-\tau} < x_t$ is equivalent to $f_t^{\text{SACP}}(x_t) < f_t^{\text{SACP}}(b_{t,t-\tau})$. However, we know from the assumption of case 2a that $f_t^{\text{SACP}}(b_{t,t-\tau}) \leq e^{-r}\mathbb{E}_t\{f_{t+1}^{\text{SACP}}(x_{t+1})\}$, and this contradicts $f_t^{\text{SACP}}(x_t) = e^{-r}\mathbb{E}_t\{f_{t+1}^{\text{SACP}}(x_{t+1})\}$. Second, $b_{t,t-\tau}$ is in the top flat area of the SACP function, in which case $f_t^{\text{SACP}}(b_{t,t-\tau}) = P_t > P_{t+1} \geq e^{-r}\mathbb{E}_t\{f_{t+1}^{\text{SACP}}(x_{t+1})\}$, and therefore, $b_{t,t-\tau} < x_t$ means $f_t^{\text{SACP}}(x_t) \geq f_t^{\text{SACP}}(b_{t,t-\tau}) > e^{-r}\mathbb{E}_t\{f_{t+1}^{\text{SACP}}(x_{t+1})\}$, and this contradicts $f_t^{\text{SACP}}(x_t) = e^{-r}\mathbb{E}_t\{f_{t+1}^{\text{SACP}}(x_{t+1})\}$. Thus, $x_t \notin (b_{t,t-\tau}, b_t)$, and according to equation (5) the only remaining choice is $x_t = b_{t,t-\tau}$.

Case 2b $f_t^{\mathbf{SACP}}(b_t) \leq e^{-r}\mathbb{E}_t\{f_{t+1}^{\mathbf{SACP}}(x_{t+1})\} < f_t^{\mathbf{SACP}}(b_{t,t-\tau}) \Rightarrow f_t^{\mathbf{SACP}}(x_t) =$ $e^{-r}\mathbb{E}_t\{f_{t+1}^{\mathbf{SACP}}(x_{t+1})\}$: Firstly, from proposition 3 we can conclude that $x_t \neq b_{t,t-\tau}$ and according to equation (5) $x_t > b_{t,t-\tau}$. Now there are two cases. First, if $f_t^{\text{SACP}}(b_t) = e^{-r}\mathbb{E}_t\{f_{t+1}^{\text{SACP}}(x_{t+1})\}$

then x_t cannot be in $(b_{t,t-\tau}, b_t)$ because based on proposition 1 this means $f_t^{\text{SACP}}(x_t) = e^{-r}\mathbb{E}_t\{f_{t+1}^{\text{SACP}}(x_{t+1})\} = f_t^{\text{SACP}}(b_t)$ and contradicts $x_t \in (b_{t,t-\tau}, b_t)$. Thus, in this case, the only remaining possibility is $f_t^{\text{SACP}}(x_t) = f_t^{\text{SACP}}(b_t) = e^{-r}\mathbb{E}_t\{f_{t+1}^{\text{SACP}}(x_{t+1})\}$. Second, if $f_t^{\text{SACP}}(b_t) \neq e^{-r}\mathbb{E}_t\{f_{t+1}^{\text{SACP}}(x_{t+1})\}$ (or equivalently $f_t^{\text{SACP}}(b_t) < e^{-r}\mathbb{E}_t\{f_{t+1}^{\text{SACP}}(x_{t+1})\}$), according to proposition 2, $x_t \neq b_t$. Thus, the only remaining possibility is that $b_{t,t-\tau} < x_t < b_t$, and according to proposition 1 this means $f_t^{\text{SACP}}(x_t) = e^{-r}\mathbb{E}_t\{f_{t+1}^{\text{SACP}}(x_{t+1})\}$. $\square$

Let us represent the oldest (newest) SREC vintage at time t by $y_t^{min} = \lceil t \rceil - \tau$ ($y_t^{max} = \lceil t \rceil$).

The following theorem represents an important property of the optimal policy and demonstrates how the optimal submission under ADAPT is chosen by market participants to balance prices through time.

THEOREM 3. *If $x_t = X_t^*(S_t)$ is an optimal policy at time $t \in \mathbb{N}$, and $x_t > b_{t,t-\tau}$ for all $t \in \mathbb{N}$, we have $e^{-r}\mathbb{E}_t\{f_{t+1}^{SACP}(x_{t+1})\} = e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t, y_t^{max}}\}$.*

Proof. At the end of their life ($T = y_t^{max} + \tau$), vintage y_t^{max} SRECs are priced at $p_{T,y_t^{max}} = f_T^{\text{SACP}}(x_T)$ according to the price formula (3). At time $T-1$, we have

$$\begin{aligned}
p_{T-1,y_t^{max}} &= \max\{f_{T-1}^{\text{SACP}}(x_{T-1}), e^{-r\Delta t}\mathbb{E}_{T-1}\{p_{T-1+\Delta t, y_t^{max}}\}\} \\
&= \max\{f_{T-1}^{\text{SACP}}(x_{T-1}), e^{-r}\mathbb{E}_{T-1}\{p_{T,y_t^{max}}\}\} \\
&= \max\{f_{T-1}^{\text{SACP}}(x_{T-1}), e^{-r}\mathbb{E}_{T-1}\{f_T^{\text{SACP}}(x_T)\}\}.
\end{aligned}$$

But according to proposition 1, because $x_t > b_{t,t-\tau}$, we have $f_t^{\text{SACP}}(x_t) \geq e^{-r}\mathbb{E}_t f_{t+1}^{\text{SACP}}(x_{t+1})$ for all t. Therefore, we obtain $p_{T-1,y_t^{max}} = f_{T-1}^{\text{SACP}}(x_{T-1})$. Continuing this argument while going backward in time, we get $p_{t+1,y_t^{max}} = f_{t+1}^{\text{SACP}}(x_{t+1})$. Therefore, we have $e^{-r}\mathbb{E}_t\{f_{t+1}^{\text{SACP}}(x_{t+1})\} = e^{-r}\mathbb{E}_t\{p_{t+1,y_t^{max}}\} = e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t, y_t^{max}}\}$. $\square$

If some banking is allowed, then it is very unlikely that at a compliance time only the oldest SRECs ($b_{t,t-\tau}$) are submitted. Specifically when τ is a number like 4 as in NJ, there are typically very few (if any) of these SRECs left and newer SRECs must be submitted instead. Under this assumption, the following theorem shows that no price difference is expected among older and newer SRECs under ADAPT.

THEOREM 4. *If $x_t = X_t^\pi(S_t)$ is an optimal policy at time $t \in \mathbb{N}$, and $x_t > b_{t,t-\tau}$ for all $t \in \mathbb{N}$, all SREC vintages have the same price.*

Proof. According to the price definition (3) and theorem 2 (when $x_t > b_{t,t-\tau}$) we have

$$\begin{aligned} p_{t,y} &= \max\{f_t^{\text{SACP}}(x_t), e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t,y}\}\} \\ &= \max\{e^{-r}\mathbb{E}_t\{f_{t+1}^{\text{SACP}}(x_{t+1})\}, f_t^{\text{SACP}}(b_t), e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t,y}\}\}. \end{aligned}$$

Replacing $e^{-r}\mathbb{E}_t\{f_{t+1}^{\text{SACP}}(x_{t+1})\}$ with its value from theorem 3 we obtain

$$p_{t,y} = \max\{e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t,y_t^{max}}\}, f_t^{\text{SACP}}(b_t), e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t,y}\}\}.$$

However, from price definition (3) we know that $e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t,y_t^{max}}\} \geq e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t,y}\}$. This simplifies the value of $p_{t,y}$ to

$$p_{t,y} = \max\{e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t,y_t^{max}}\}, f_t^{\text{SACP}}(b_t)\}.$$

□

In other words, a price difference can only emerge if the optimal submission decision x_t, needed to balance today's penalty with expected future prices, would fall below the bound $b_{t,t-\tau}$. In this case, it is instead preferable to submit $b_{t,t-\tau}$ to avoid wasting SRECs, thus bringing down the price of the expiring vintage compared to others.

According to this general market design, instead of a high penalty of P_y for $x_y = R_y - \varepsilon$, and a low penalty of zero for $x_y = R_y + \varepsilon$ in the current mechanism, a middle ground is chosen in which the penalty of both of these cases is near $\frac{P_y}{2}$. However, this does not mean that the total penalty payment under both regimes remains the same in expectation. The following theorem shows that the total penalty payment is always more under the sloped market design.

THEOREM 5. *Total penalty payment of the sloped market design is always greater than or equal to the original step mechanism for any fixed number of submitted SRECs.*

Proof. Let x_t denote the total submitted SRECs at time $t \in \mathbb{N}$. Also let us represent the total penalty payment in the current cliff mechanism and the sloped mechanism by $h_t^{\text{Cliff}}(x_t)$ and $h_t^{\text{ADAPT}}(x_t)$ respectively. We also represent the SACP functions of the ADAPT and cliff policies with $f_t^{\text{ADAPT}}(x_t)$ and $f_t^{\text{Cliff}}(x_t)$ respectively.

a_0	a_1	a_2	a_3	a_4	a_5	a_6	σ
10.9558	-0.1209	0.0900	0.2151	0.3859	-0.0151	1.27×10^{-3}	0.186

Table 2 Estimated parameters for our linear model

According to our definition of the penalty functions, $\int_0^\infty f_t^{\text{ADAPT}}(z)dz = \int_0^\infty f_t^{\text{Cliff}}(z)dz$, and we use K to denote this value. Hence, we have

$$h_t^{\text{ADAPT}}(z) = K - \int_0^{x_t} f_t^{\text{ADAPT}}(z)dz,$$
$$h_t^{\text{Cliff}}(z) = K - \int_0^{x_t} f_t^{\text{Cliff}}(z)dz.$$

Therefore, to prove the theorem, we must show that $\int_0^{x_t} f_t^{\text{ADAPT}}(z)dz \le \int_0^{x_t} f_t^{\text{Cliff}}(z)dz$ for any $x_t \in [0,\infty)$.

If $0 \le x_t \le R_t$, we have $0 < f_t^{\text{ADAPT}}(x_t) \le f_t^{\text{Cliff}}(x_t)$, and hence we conclude $\int_0^{x_t} f_t^{\text{ADAPT}}(z)dz \le \int_0^{x_t} f_t^{\text{Cliff}}(z)dz$.

If $x_t \ge R_t$, we have $\int_0^{x_t} f_t^{\text{Cliff}}(z)dz = K = \int_0^\infty f_t^{\text{ADAPT}}(z)dz \ge \int_0^{x_t} f_t^{\text{ADAPT}}(z)dz$. $\square$

6. Calibrating and Solving the Models

To gain insight into the effects of different SREC policies, we need to first be able to estimate some parameters for the current market design to serve as our benchmark. We can then modify some of these parameters to design new markets and assess their performance.

To represent the current market mechanism, for parameters $\tilde{R}_t$ and P_t we use the values given in table 1, which are based on the latest regulation changes in NJ. Also, we use a constant interest rate of 2% estimated based on historical interest rates (the interest rate has little effect on the results of our experiments). The current SREC market design is characterized by a sharp cliff. Therefore, we set $\lambda = 0$ to represent the current SREC market design. In addition, we set $\alpha = 0$ to capture the fact that the current market targets are nonadaptive.

6.1. Generation model parameters

The parameters for the general model are obtained by fitting a linear model to the log of the historical generation data (see Coulon et al. (2013) for more details). These parameters are given in table 2.

Also, according to the historical generation data, a normal distribution with mean zero and standard deviation 0.186 seems to characterize the noise ε_t very well.

6.2. Computing the price surface

The original dynamic programming model is computationally intractable due to the dimensionality of the state variable. However, we can reduce the dimensionality, preserving the structure and behavior of the problem yet producing a model that can be solved exactly.

Specifically, the number of dimensions can be reduced greatly by using the scalar b_t, giving the aggregate number of banked SRECs, instead of the vector $(b_t)_t$ which captures the banking by vintage. These variables only appear in the function $f_t^{\mathrm{SACP}}(x_t) = \min\{f_t^{\mathrm{SACP}}(b_{t,t-\tau}),$ $\max\{f_t^{\mathrm{SACP}}(b_t), e^{-r}\mathbb{E}_t\{f_{t+1}^{\mathrm{SACP}}(x_{t+1})|x_t\}\}$, and are only effective if the number of older SRECs that are about to expire is very large. This is quite unlikely to happen in reality (in fact, for $\tau = 4$, $b_{t,t-\tau} = 0$ is virtually guaranteed), specifically because the oldest SRECs are the first to be submitted at each compliance time (theorem 1). Therefore, when generating our price surfaces, we fit the value function around the scalar b_t, but we retain the vector of banked SRECs by vintage, $(b_t)_t$, as we simulate forward, making it possible to check to see if our approximation is valid. The transition function for b_t, then, is given by

$$b_{t+\Delta t} = b_t + g_t \Delta t - x_t.$$

The previously discussed assumption that $b_{t,t-\tau} < x_t$, means that $f_t^{\mathrm{SACP}}(x_t) = \max\{f_t^{\mathrm{SACP}}(b_t),$ $e^{-r}\mathbb{E}_t\{f_{t+1}^{\mathrm{SACP}}(x_{t+1})\}$ (see theorem 2). As $b_{t,t-\tau} < x_t$, according to theorem 3, $f_t^{\mathrm{SACP}}(x_t)$ can be determined based on expected future SREC prices of the newest vintage at time t:

$$f_t^{\mathrm{SACP}}(x_t) = \max\{f_t^{\mathrm{SACP}}(b_t), e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t, y_t^{max}}\}\}.$$

Thus, the optimal x_t can be obtained from

$$x_t = \min\{b_t, (f^{\mathrm{SACP}})_t^{-1}(e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t, y_t^{max}}\})\} \tag{6}$$

at time $t \in \mathbb{N}$. Note that $e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t, y_t^{max}}\}$ is in the sloped area of f_t^{SACP} (because P is decreasing in time) and therefore $(f^{\mathrm{SACP}})_t^{-1}$ is defined. This means we can solve our dynamic program based on SREC prices instead of the original objective function (i.e. social welfare). This simplifies calculations at each time step and directly outputs SREC prices.

According to the definition of SREC prices (equation (3)), we obtain,

$$p_{t,y} = \exp(-r\Delta t)\mathbb{E}_t[p_{t+\Delta t,y}], \qquad t \notin \mathbb{N}, \tag{7}$$

and at the compliance dates ($t \in \mathbb{N}$), we have

$$p_{t,y} = \max\{f_t^{\text{SACP}}(x_t), \exp(-r\Delta t)\mathbb{E}_t[p_{t+\Delta t,y}]\}, \qquad t \in \mathbb{N}. \tag{8}$$

We can use equations (6), (7), and (8) to compute the SREC prices using backward induction on a discretized state space.

Also, in most of our experiments, we reduce $\bar{p}_t$ to the current price p_t (by choosing $\delta = 1$ and $\gamma = 0$), in order to decrease the number of dimensions in the dynamic program and increase computational performance. As discussed and shown by example in Coulon et al. (2013), this does not have a significant impact on the results of our experiments.

We compute the price surface according to the following algorithm.

1. Discretize the state variable $S_t = (b_t, \hat{g}_t, s_t)$ on a grid ranging over $[0, \kappa^b R_{y+\tau}], [0, \kappa^g R_{y+\tau}]$, and $[-\kappa^s R_{y+\tau}, \kappa^s R_{y+\tau}]$ respectively for each vintage year y, where κ^b and κ^g are coefficients greater than one (e.g. 1.5), and κ^s is a positive number (e.g. 0.5). Also discretize the distribution of the noise variable ε_t into n outcomes $\varepsilon_{t,i}$ with probability $pr\{\varepsilon_{t,i}\}$, for $i = 1, \ldots, n$.
2. Initialize the dynamic program by calculating $p_{T,y^{fin}} = f_T^{\text{SACP}}(x_T)$ at all grid points, where y^{fin} denotes the final SREC vintage in our simulation, and $T = y^{fin} + \tau$.
3. Go backward through time and compute prices of all (possibly existing) SREC vintages $(y_t^{min}, \ldots, y_t^{max})$ from (7) and (8) at each grid point $(b_t, \hat{g}_t, s_t)$:
   (a) If $t \notin \mathbb{N}$, $\tilde{x}_t = 0$, otherwise find $\tilde{x}_t$ by solving $f_t^{\text{SACP}}(\tilde{x}_t) = e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t, y_t^{max}} | \tilde{x}_t\}$:
      i. Set $\tilde{x}_t = (1+\lambda)R_t$; at this point $f_t^{\text{SACP}}(\tilde{x}_t) = 0$.
      ii. $x_t = \min\{\tilde{x}_t, b_t\}$
      iii. $p_{t,y_t^{max}} = \max\{f_t^{\text{SACP}}(\tilde{x}_t), f_t^{\text{SACP}}(b_t)\}$.
      iv. From (1) $\hat{g}_{t+\Delta t} = \hat{g}_t \exp(a_5 \Delta t + a_6 \bar{p}_t \Delta t)$; if $\delta = 1$, $\bar{p}_t = p_{t,y_t^{max}}$.
      v. For each $i = 1, \ldots, n$ from (2)

$$\begin{aligned} g_{t+\Delta t,i} = \hat{g}_{t+\Delta t} \exp\Big(&a_1 \sin(4\pi(t+\Delta t)) + a_2 \cos(4\pi(t+\Delta t)) \\ &+ a_3 \sin(2\pi(t+\Delta t)) + a_4 \cos(2\pi(t+\Delta t)) + \varepsilon_{t,i}\Big). \end{aligned}$$

vi. $b_{t+\Delta t,i} = b_t + g_{t+\Delta t,i}\Delta t - x_t$.

vii. $s_{t+\Delta t} = b_t - \tilde{R}_t$.

viii. Using the discretized distribution function

$$\mathbb{E}_t[p_{t+\Delta t,y_t^{max}}] = \sum_{i=1}^{n} pr\{\varepsilon_{t,i}\} p_{t+\Delta t,y_t^{max}}\Big(b_{t+\Delta t,i}, \hat{g}_{t+\Delta t}, s_{t+\Delta t}\Big).$$

ix. If $f_t^{\text{SACP}}(\tilde{x}_t) < e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t,y_{t+\Delta t}^{max}}\}$, update $\tilde{x}_t$ to $\tilde{x}_t - k$ with $k > 0$, and go to (ii).

(b) Set $x_t = \min\{b_t, \tilde{x}_t\}$, and compute $b_{t+\Delta t}$, $\hat{g}_{t+\Delta t}$, and $s_{t+\Delta t}$ as of (a iii) to (a vi) equations.

(c) For $y : y_t^{max}$ to y_t^{min}

i. Calculate $\mathbb{E}_t[p_{t+\Delta t,y}] = \sum_{i=1}^{n} pr\{\varepsilon_{t,i}\} p_{t+\Delta t,y}\Big(b_{t+\Delta t}, \hat{g}_{t+\Delta t}, s_{t+\Delta t}\Big)$.

ii. $p_{t,y} = \max\{f_t^{\text{SACP}}(x_t), e^{-r\Delta t}\mathbb{E}_t\{p_{t+\Delta t,y}\}\}$.

CPU times for solving a simulation over a decade to appropriately tackle accuracy, with a single fine-grained grid, are excessive (in the order of several weeks). As our state variables (e.g. generation) lie on different ranges for early and late years, we can use separate smaller grids for different vintage years (as given in step 1 above) instead of a single large grid. This maintains the same level of accuracy, while keeping the problem computationally tractable. Experiments were run for 50 to 100 grid points for each state variable for each vintage year, and this was found to produce an acceptable tradeoff between accuracy and CPU times.

7. Experiments

We now report on experiments designed to provide insights into the effect of different market designs. We begin our experiments assuming a steady growth in targets, a strategy that ensures that SREC prices do not rapidly fall towards zero. For this purpose, we assume that requirements are determined by $R_y = \alpha \exp \beta(y - 14)$, with α equal to the requirement level of EY2014 and β equal to a positive number ($\beta = 0.35$ in the following experiments). Also, we assume five years of lifetime ($\tau = 4$). After studying the behavior of market designs using this formula for requirements, we report on a study of the current requirement schedule.

7.1. The slope of the SACP function

There are four critical differences between ADAPT and the current market mechanism. Firstly, if λ is large enough (producing a shallow SACP function), extreme prices of zero or P_t only

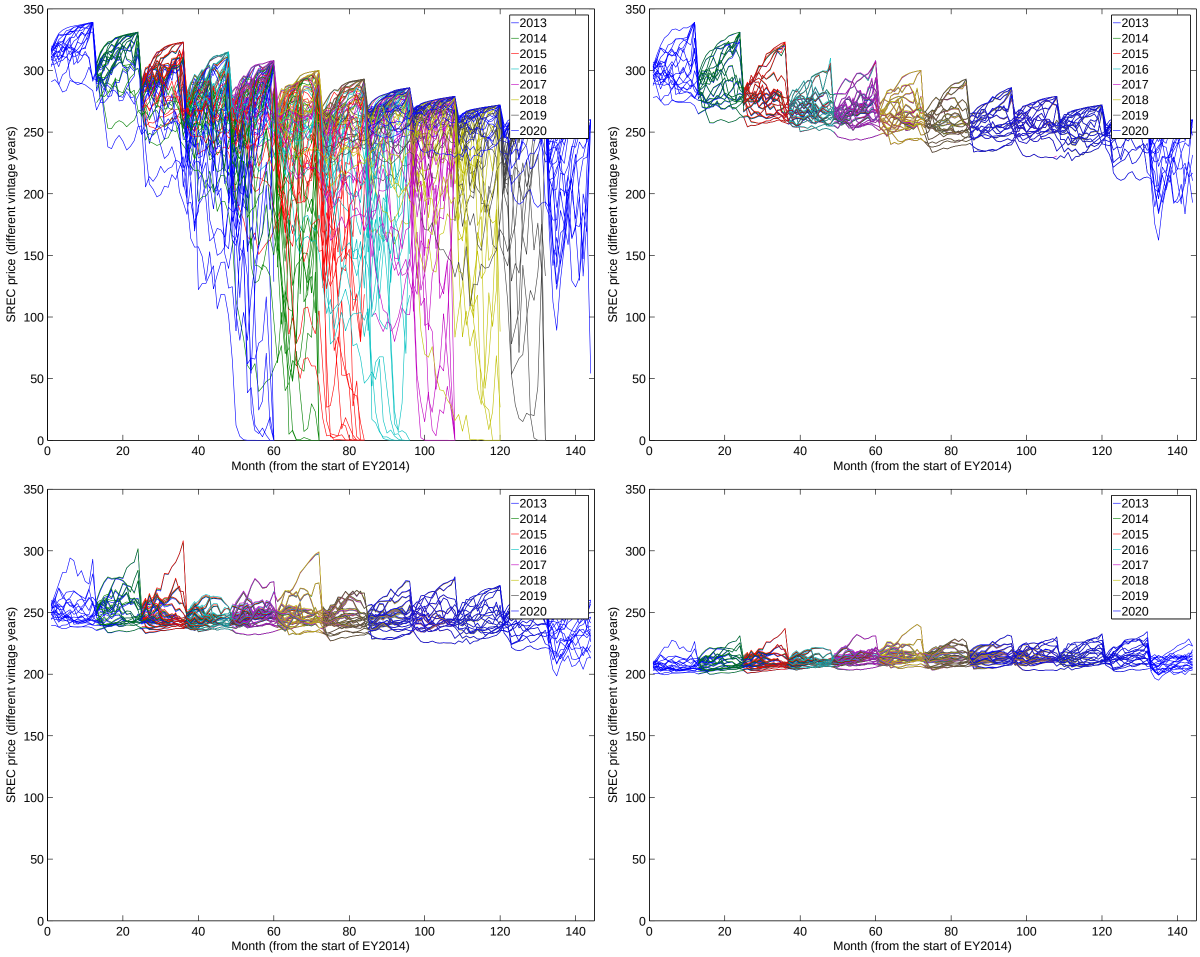


Figure 4 20 price sample paths for an 8 year simulation (for $\lambda = 0$, 0.1, 0.3, and 0.7 from left to right, top to bottom).

happen if the total banked SRECs are either very high or very low. Secondly, in the sloped mechanism, penalties can have any values between 0 and P_t unlike the binary nature of the step mechanism. Thirdly, in the step market design, if the total banked SRECs are less than the requirement, they must all be submitted, as P_t is a decreasing sequence through time and so the current penalty is more than any possible future price. Thus, in effect there is no real choice for how many SRECs to submit and how many to bank in the step mechanism, while under ADAPT participants can choose the submission value to balance prices before and after submission. Finally, the second and third differences result in equal prices across vintages under ADAPT.

Figures 4 and 5 compare SREC prices of different vintage years between a step SACP mechanism and a sloped SACP mechanism. These results are obtained from twenty sample paths,

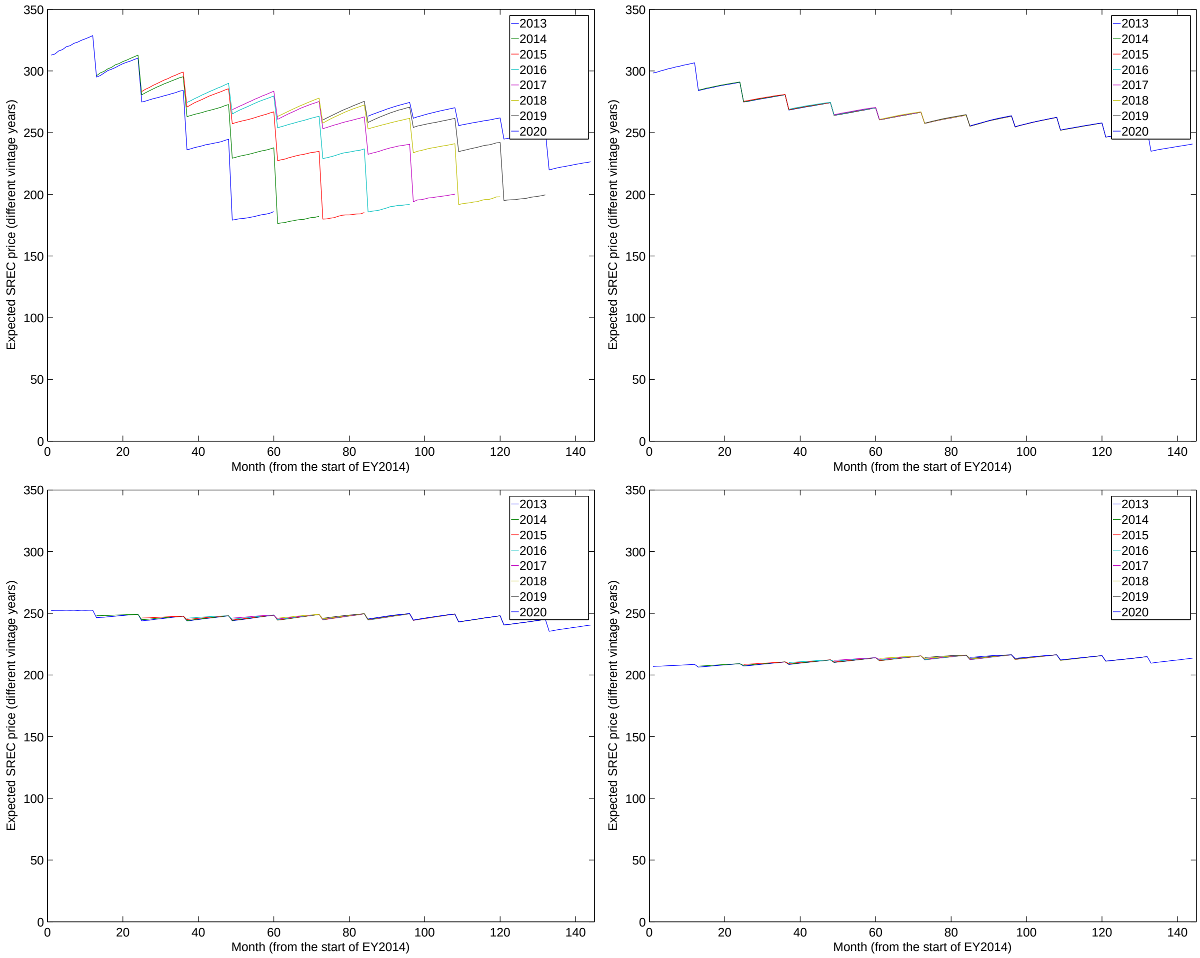


Figure 5 Average SREC prices over 10000 simulations (for $\lambda = 0, 0.1, 0.3,$ and 0.7 from left to right, top to bottom)

and an average of 10000 simulations with 100 grid points for each state variable $\hat{g}_t$ and b_t (the price feedback parameter a_6 is 7×10^{-4}). To obtain a meaningful comparison in which price volatility can be easily observed, we need prices that are not too high or too low. Therefore, we do not match our initial conditions to the market today, and instead we use $\hat{g}_{13} = R_{14} \exp(-\beta)$ and $b_{13} = 0$ which result in mid-range prices.

Figures 4 and 5 show that the cliff policy (with $\lambda = 0$) not only produces lower prices moving forward, but also far more volatile prices. Figure 5 also shows that there is no price difference between SRECs of different vintages when we used a sloped SACP ($\lambda > 0$), a conclusion that appears to be valid for all positive values of λ. This observation matches our expectation based on theorem 4. Both of these observations indicate that the sloped mechanism provides a much safer and more attractive environment for investment in solar power.

Also, these figures show that price volatility decreases when λ increases. By using higher λs, we effectively increase the total requirement level. Higher requirement levels mean less of a chance of hitting zero and so higher and more stable prices. In ADAPT, (in most cases) an equilibrium can form between the price of SRECs before and after compliance. This is because if the price after compliance is expected to be higher, a firm would prefer to bank more, accepting a higher penalty today while reducing the price of SRECs in the future, until these two values are balanced.

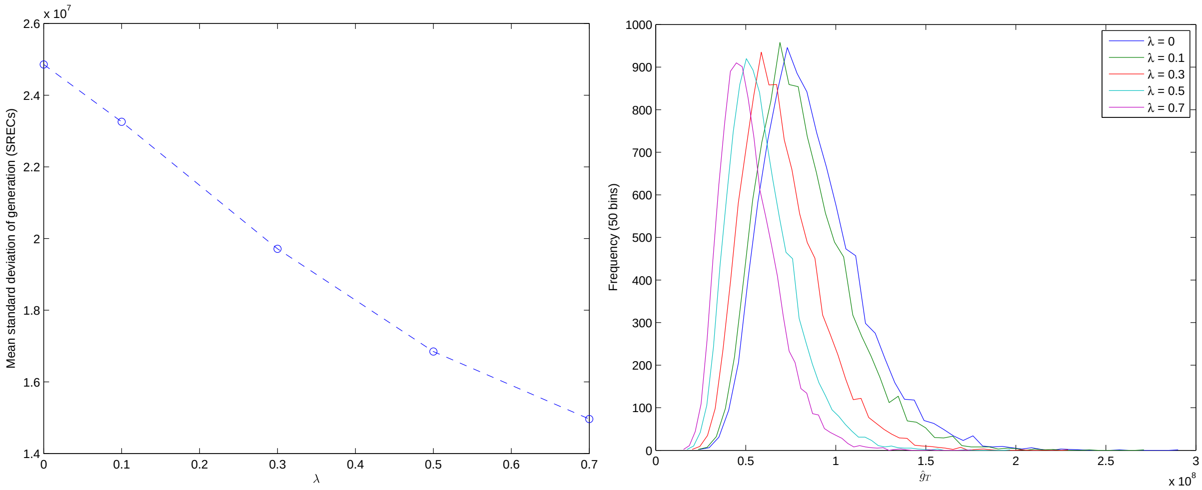


Figure 6 Left: Mean standard deviation of generation across different sample paths for different λs. Right: Distribution (discretized frequency) of installed capacity for different λ values.

One may view increasing λ as moving from a quantity-based regulatory policy towards a price-based policy, and thus expect that increasing λ will decrease the probability of meeting the requirement. The right plot in figure 6 shows that higher levels of λ reduce the final installed capacity across 10000 sample paths, which means higher λs decrease the probability of meeting the target. The figure on the left, however, shows that generation variability decreases across different sample paths, when λ increases. This can be explained by the previously observed behaviour of higher slopes in decreasing price volatility. As discussed earlier (e.g. see equations (1) and (2)), generation is a function of SREC prices, and thus it is not surprising that a market design yielding lower price volatility, in comparison with others, decreases generation volatility as well.

However, increasing λ is equivalent to increasing the requirement, and therefore, imposing a higher total payment to the society (as theorem 5 suggests). Therefore, one may seek a balance between higher payments and less volatility. It seems that the main goal of making solar energy more attractive for investment can be achieved by a market mechanism that produces relatively high prices with no price difference between various vintages. An ADAPT mechanism with a small positive λ can provide these major benefits, and it seems that price variability for small λs does not produce a big risk for investors. Therefore, one may argue that an ADAPT mechanism with a small positive λ is an appropriate choice for improving market attractiveness for investors and simultaneously imposing a lower cost on market participants.

7.2. Adaptive requirements

A challenge faced by policy makers is the design of a fixed target schedule, which appears to require estimating a highly uncertain rate of market adoption many years in advance. Such a strategy is based on a forecast of behaviour, and is not adaptable to many different outcomes, such as the possibility of changes in the cost of installing solar and the behavior of the market. We can circumvent this problem by allowing the requirements to adapt to the current generation level.

In our model, we use a combination of a fixed rule and an adaptive rule:

$$R_y = \tilde{R}_y + \alpha\ (b_{y-1} - \tilde{R}_{y-1}), \qquad 0 \leq \alpha \leq 1, y \in \mathbb{N}.$$

According to this mechanism, there are two sets of requirements. The base requirement level $\tilde{R}_y$ is a fixed number determined by the regulators and known to market participants, and the adaptive requirement R_y is the effective requirement for energy year y. According to this rule, a portion of last year surplus (shortage) is added to (deducted from) the already known base requirement $\tilde{R}_y$. Note that surplus and shortage is defined based on the difference between the total number of available SRECs and the base requirement. One may expect that such an adaptive mechanism can yield less volatile prices by enforcing a balanced growth for generation and requirement levels.

As discussed earlier in section 4, modeling a market design with adaptive requirements requires one extra dimension in our dynamic program. Although this extra dimension increases

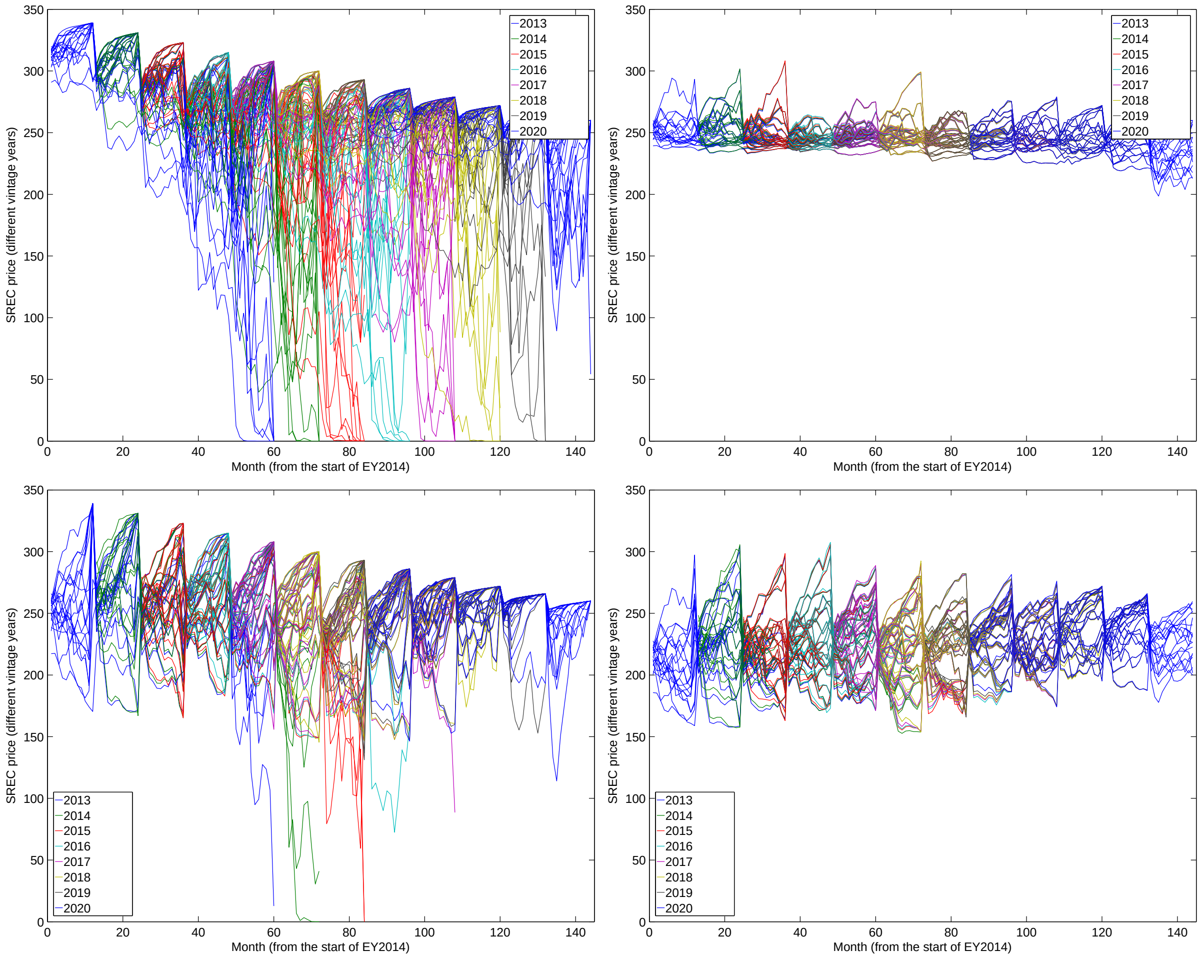


Figure 7 20 price sample paths for an 8 year simulation (first row: non-adaptive requirements; second row: adaptive requirements; left: cliff policy; right: sloped policy; for (λ, α) tuples (0,0), (0.3,0), (0, 0.5), and (0.3,0.5) from left to right, top to bottom).

the computational time significantly (from around one hour to around a week with 50 grid points for each dimension), we still are able to solve this problem using an exact dynamic programming approach. Figures 7 and 8 compare the results of the non-adaptive mechanism (for (λ, α) equal to (0,0) and (0.3,0)) with a similar mechanism with adaptive requirements ((λ, α) equal to (0,0.5) and (0.3,0.5)). According to these results, an adaptive cliff policy reduces price volatility and maintains higher SREC prices in comparison with a simple cliff policy (the current mechanism). Adaptive requirements, however, increase price volatility in comparison with a simple sloped policy (ADAPT with $\lambda > 0$, and $\alpha = 0$), and therefore are not desirable in this case.

Figure 9 shows SREC prices for 20 sample paths of another experiment on a sloped policy ($\lambda = 0.3$) with and without adaptive requirements ($\alpha = 0$, $\alpha = 0.5$, and $\alpha = 1$). In this experiment

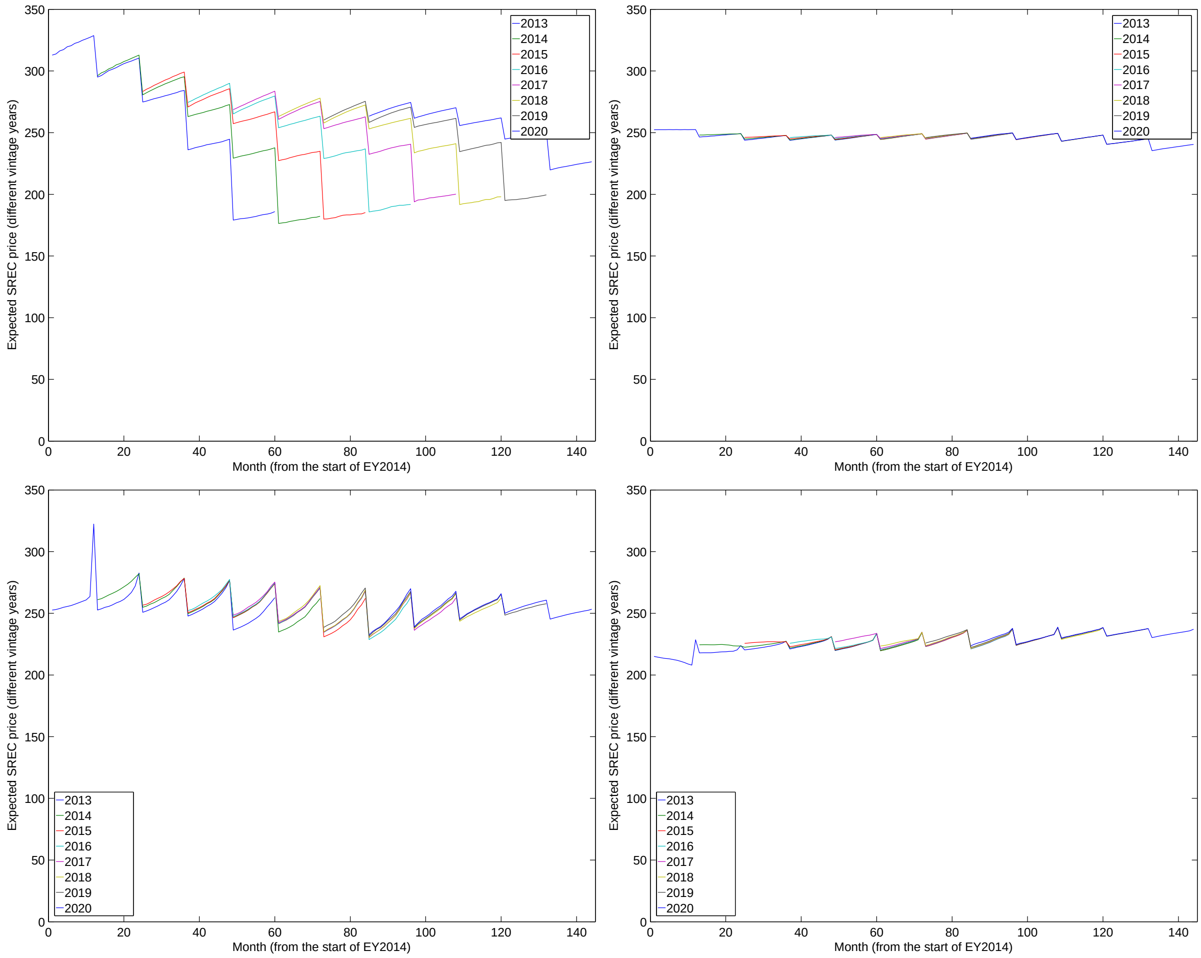


Figure 8 Average SREC prices over 10000 simulations (first row: non-adaptive requirements; second row: adaptive requirements; left: cliff policy; right: sloped policy; for (λ, α) tuples (0,0), (0.3,0), (0, 0.5), and (0.3,0.5) from left to right, top to bottom).

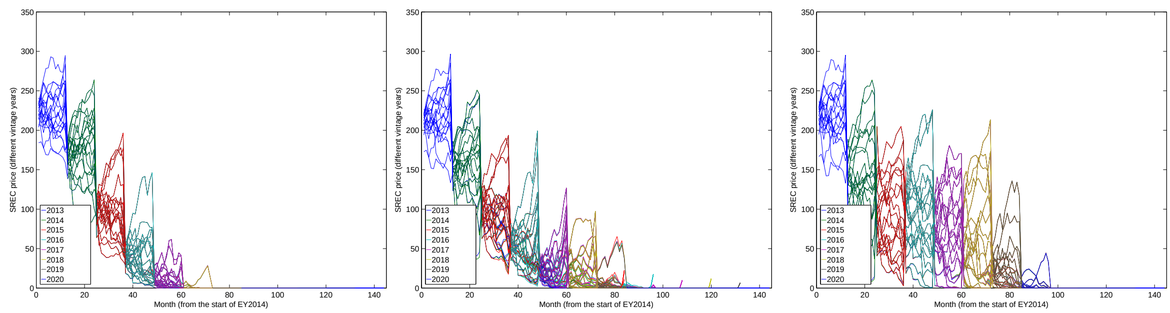


Figure 9 20 sample paths for original requirements for $\alpha = 0$, 0.5, and 1 respectively ($\lambda = 0.3$, $\hat{g}_{13} = R_{14}\exp(-\beta)$)

the original requirement schedule is used, and thus the rate of generation growth is not in line with the rate of requirement growth. The adaptive requirement mechanism ($\alpha = 0.5$) delays the rate of convergence to zero for a couple of years, but it cannot completely stop it. This can be better explained if we refer to the case with $\alpha = 1$. When $\alpha = 1$, any value of positive surplus would be added to the next year requirement, and this would somewhat neutralize the effect of banked SRECs. However, the rate of generation is still more than the requirement and thus prices fall to zero but slower in comparison with the cases of $\alpha = 0$ and 0.5. The price variability increases perhaps because adaptivity counteracts the effect of banking, and as we discuss in the next section, lack of banking possibility can increase price volatility.

All in all, although the adaptive requirements can increase price volatility, they can also, to some extent, correct human mistakes in predicting the rate of generation and ensure more balanced prices in the long term, without the need for frequent regulatory fixes.

7.3. More banking years

Another strategy chosen by the regulators during the life time of the NJ SREC market has been increasing the number of banking years. This strategy firstly ensures higher SREC prices as a result of extra years of validity. Secondly, this can stabilize prices from falling too low or rising too high, by striking a balance among different energy years.

Figure 10 shows the results of an experiment on the number of banking years. Note that at each time, different SREC vintages may exist, and prices (e.g. price means) are all represented with the same color; however, the lines with higher prices correspond to newer vintages. According to this figure, the difference between the 10th and the 90th percentile of SREC prices of the newer vintages, which are the majority of SRECs at each time, decreases with higher number of banking years. For example, with five years of lifetime, prices of the majority of SRECs (zero or one year old SRECs) are much less volatile than those of one or two years of lifetime.

7.4. Rate of requirement growth

The early years of New Jersey's SREC market has seen an exponential growth in generation as the market responded to the generous incentives provided by high SREC targets and penalties. However, according to the latest regulation changes in 2012 (as shown in table 1), the current rate of requirement growth per year is a decreasing function of time (R is a concave function of time). This means that although the current requirement levels are compatible with the

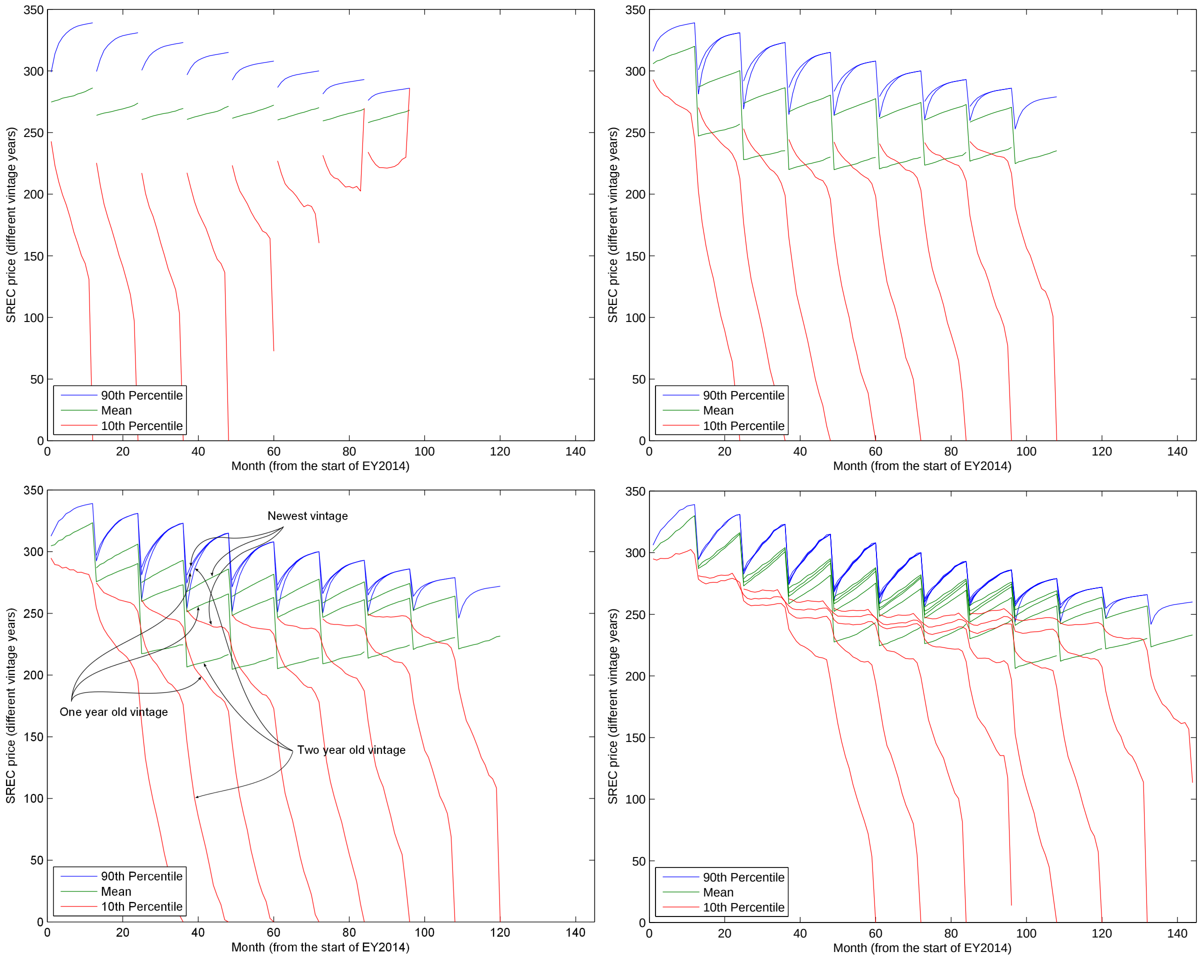


Figure 10 The 10th and the 90th percentiles and mean of SREC prices for 10000 simulations with 1, 2, 3, and 5 years of lifetime from left to right top to bottom respectively (with $\lambda = 0, \alpha = 0$).

generation level for a few years in the future, there is a very good chance that the generation rate exceeds the requirement level afterwards. For example, SREC generation, as we write in July 2014 with 1,189,542 SRECs issued for EY2014 has greatly exceeded the old requirement of 772,000 SRECs for EY2014, and is much more than the old SREC requirement of EY 2016 (1,150,000). Indeed, this was the main motivation for the recent rule changes in the NJ SREC market (i.e. increasing requirement levels to prevent prices from falling to zero).

The planned growth in requirements represents an important policy decision. Managing the rate of growth of the solar industry requires striking a balance between the goal of growing the number of installations, and increasing the cost to the society as a result of solar subsidies.

Figure 11 provides a comparison between SREC prices under the current requirement levels and an exponentially growing requirement scheme based on 10000 simulations. In this example,

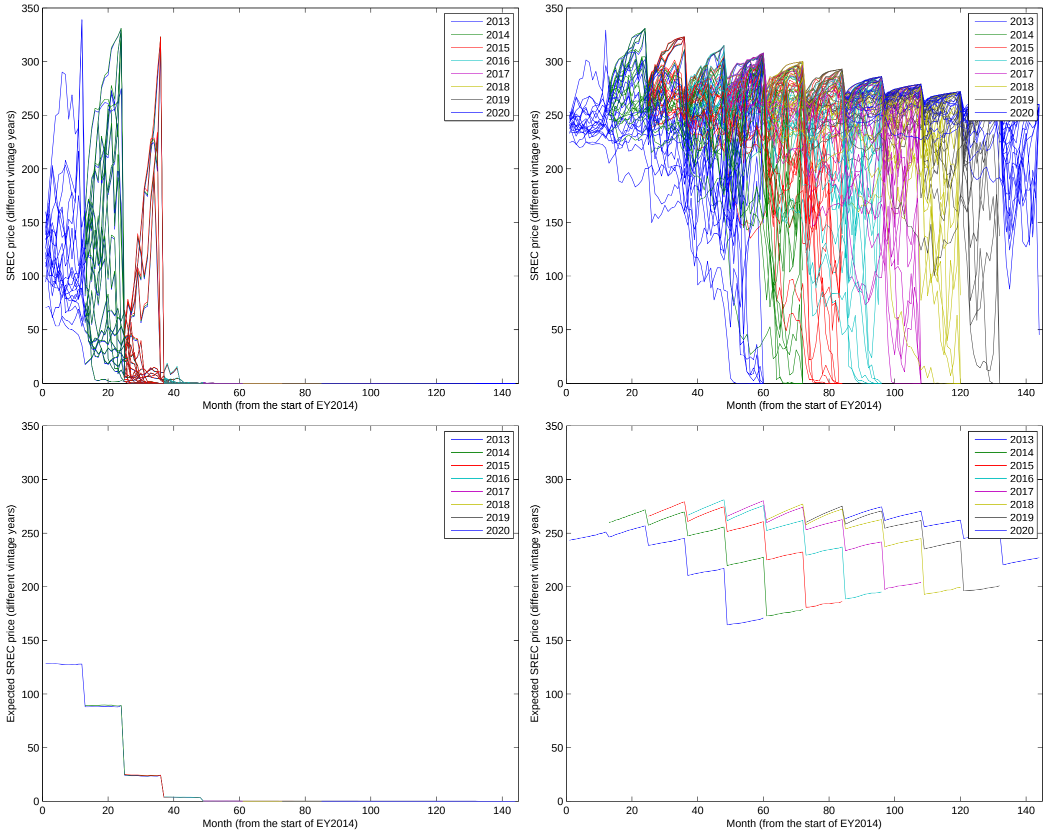


Figure 11 SREC prices under the current requirement scheme (left) vs. an exponentially growing scheme (right) for 20 identical sample paths, and average of 10000 sample paths.

the exponentially growing requirements $R'_t = \alpha \exp(\beta(t-14))$ is used, where $t = 14$ corresponds to end of EY2014, $\alpha = R_{14}$ and $\beta = 0.35$. According to this example, with the current requirements, SREC prices are expected to fall to zero after three years of decreasing prices. However, prices may actually stay non-negative (but low) due to market expectations of any possible regulation changes. Also, notice that there is no price difference between different vintage years under the current requirements. This is because all SRECs will be used up before their final expiry date and therefore, newer SRECs do not provide any advantage to the older SRECs.

Figure 9 provides 20 sample paths for a sloped mechanism with original requirements and $\alpha = 0$ and 0.5 respectively ($\lambda = 0.3$, $\hat{g}_{13} = R_{14}\exp(-\beta)$).

Note that although not necessarily realistic, exponentially growing requirements are consistent with the assumptions of our generation model, and therefore useful for the purpose of running experiments in which prices remain high and relatively stable. Despite the current requirements, exponentially growing requirements can guarantee long-term relatively high prices for our generation model, and so can provide a good environment for comparing different policy options.

8. Conclusion

The current SREC market design produces volatile prices and this reduces the attractiveness of this market for investment. This itself creates other problems such as reducing competitiveness of the market and therefore inefficiency in SREC generation. We propose the ADAPT policy which uses a sloped SACP function. We develop a dynamic programming model in order to predict collective market behaviour, and we derive and prove some of the properties of this market such as the collective SREC submission policy. We then use this submission policy and other market properties in a pricing model that enables us to compare the performance of different market designs in terms of price level and volatility.

The results of our experiments show that the ADAPT policy (specifically the sloped SACP function) can greatly reduce price drops and volatility, and ensure relatively high and stable prices in long-term.

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