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SMART-SREC: A stochastic model of the New Jersey solar renewable energy certificate market

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ABSTRACT

Markets for solar renewable energy certificates (SRECs) are gaining in prominence in many states, stimulating growth of the US solar industry. However, SREC market prices have been extremely volatile, causing high risk to participants and potentially less investment in solar power generation. Such concerns necessitate the development of realistic, flexible and tractable models of SREC prices that capture the behavior of participants given the rules that govern the market. We propose an original stochastic model called SMART-SREC to fill this role, building on established ideas from the carbon pricing literature, and including a feedback mechanism for generation response to prices. We calibrate the model to the New Jersey market and backtest it, analyzing parameter sensitivity and demonstrating its ability to reproduce historical dynamics. Finally, we run simulations to investigate the role and impact of regulatory parameters, thus providing insight into the crucial role played by market design.

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Introduction

While cap-and-trade schemes for carbon emissions have gained widespread attention in recent years as market-based tools for implementing environmental policy, an alternative approach which is now growing rapidly in many regions is the use of 'renewable energy certificates' or RECs (often called 'green certificates' or GCs in Europe). For each MWh of renewable energy produced, a tradable certificate is issued to the generator who can then sell this REC in the marketplace to a load-serving entity (electric utility) that is subject to the annual requirement on the percentage of its electricity procured from renewables. If desired, these markets can be geared specifically towards encouraging growth in a particular type of renewable energy, as in the case of the New Jersey (NJ) market for SRECs (solar renewable energy certificates). This market provides an excellent example for us to study, as the state of New Jersey (NJ) has witnessed dramatic growth from under 10 MW of solar installations when SRECs were first issued in 2005 to over 950 MW of installations by the end of 2012 (NJCleanEnergy, 2015). Similar markets exist across the US and around the world, including several European countries (eg, UK, Italy, Belgium, Sweden, Norway), Australia and India. In the absence of a national carbon emissions markets in the US (and given recent challenges faced by an oversupplied European carbon market), REC markets are emerging as an important policy alternative to putting a price on emissions directly, and consequently a potentially valuable tool in fighting climate change.

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Environmental markets

Economic instruments for achieving environmental goals are classically categorized as either price-based or quantity-based, depending on which of these two variables is chosen by a regulator. Price-based instruments such as taxes, subsidies and feed-in tariffs fix a price, while markets for tradable certificates involve a chosen quantity target via a cap or requirement. There is a natural tradeoff between price and quantity certainty, since by fixing one of them, the other becomes uncertain and uncontrollable, subject to fluctuations in supply or demand. There is much literature debating the merits of these two alternatives (see for example [Weitzman, 1974](#); [Stavins, 1996](#); [Moledina et al., 2003](#); [Hepburn, 2006](#)), and we do not aim to enter this debate here. Instead, we construct a model to investigate the middle ground by analyzing some ways in which quantity-based trading schemes can lead to more stable prices, without giving up the trading scheme all together. This viewpoint is certainly not new, as various hybrid schemes (such as price collars and strategic reserves of allowances) have been proposed in the carbon literature, including by [Gruell and Taschini \(2011\)](#) and [Hepburn \(2006\)](#). Indeed, the relatively short histories of both REC and carbon markets are littered with examples of regulators responding frequently to undesirable price swings with artificial market fixes and adjustments. SREC markets in particular have already witnessed many different designs, redesigns and rule changes in various states, with a clear interest in producing more stable prices to encourage and support investments in renewables.¹ Although price volatility is a natural feature of any quantity-based mechanism, excessive price instability can potentially have a chilling effect on the development of a healthy solar industry, as investors react to the risk that SREC prices will not support their total investment or their ongoing financing costs.

It is well known that environmental markets for traded certificates are intrinsically susceptible to unstable prices, which can potentially swing rapidly from nearly zero to the penalty level, despite relatively small changes in the underlying supply and demand forces. Various papers discuss this problem for SREC markets in particular and describe how it stems from the artificial vertical demand curve imposed by standard market design (see [Felder and Loxley, 2012](#); [Berry, 2002](#); [Kildegard, 2008](#); [Marchenko, 2008](#)). However, this branch of the literature stops short of building a stochastic price model, which is needed to fully understand the dynamics of SREC prices, their sensitivity to regulatory parameters and their implications on market participant behavior. Such issues are of utmost importance both for individuals or businesses considering investments in new solar installations and for utility companies managing their existing SREC price risk. Furthermore, a realistic model for SREC prices based on market structure can shed much-needed light on key policy challenges related to the design of these young and still developing markets, thus helping to ensure their continued success in the future.

Existing literature on REC prices

Despite the clear need for innovative new models to describe SREC price behavior, there exists very little academic literature on the topic. A number of government-sponsored reports provide useful overviews of the market and some general discussion of factors affecting price dynamics.² Historical SREC price and issuance data are also easily available online from websites such as [NJCleanEnergy \(2015\)](#), [FlettExchange \(2015\)](#), and [SRECtrade \(2015\)](#). Nonetheless, there exist very few attempts to build stochastic models which describe how market rules affect price behavior. [Amundsen et al. \(2006\)](#) provide an early proposal for modeling green certificate prices in Europe, building on the classical commodity storage models of [Deaton and Laroque \(1996\)](#), [Routledge et al. \(2000\)](#) and others, equating the banking of certificates for future years to the storage of grains or metals. However, they do not incorporate into their work the regulatory structure of the market and its unique features. Several recent economic analyses of European green certificate markets are available, but these focus primarily on evaluating the success of existing mechanisms, comparing with feed-in tariffs, and discussing future prospects (see for example [Aune et al., 2012](#); [Haas et al., 2011](#); [Tamas et al., 2010](#)). In contrast, we shall focus exclusively on understanding and modeling price dynamics, incorporating relationships with all important features of market structure and key fundamental price drivers.

In contrast to REC markets, many approaches exist for describing equilibrium price formation and dynamics in markets for emission allowances, a prominent topic in the field of environmental economics. The literature dates back several decades to early work such as [Montgomery \(1972\)](#) showing how cost minimization can produce an equilibrium price for a pollution credit. [Rubin \(1996\)](#) follows with an analysis of the inter-temporal effects of banking and borrowing credits between periods in a deterministic model. More recently, [Carmona et al. \(2010\)](#) present a very general stochastic framework for the behavior of electricity and carbon market participants, leading to a single-period equilibrium allowance price given by the penalty price times the probability of a shortage of credits at the compliance date (relative to actual pollution). This same pricing formulation appears throughout other recent models (cf. [Seifert et al., 2008](#); [Howison and Schwarz, 2012](#);

¹ Such considerations are visibly at the forefront of ongoing discussions, as seen for example by a March 2014 draft report on solar market volatility, produced for the New Jersey Board of Public Utilities (BPU) and made publically available via [NJCleanEnergy \(2015\)](#). In this draft report, various long term policy options for mitigating price volatility are discussed and evaluated, including price floors, the use of auctions, and supply-responsive demand formulas.

² See for example [Wiser et al. \(2010\)](#) and [Bird et al. \(2011\)](#), both sponsored by the Department of Energy's National Renewable Energy Laboratory, for summaries of the development of SREC markets across the US through 2011, or alternatively the New Jersey Clean Energy Program's annual reports and regularly updated news on the New Jersey market at [NJCleanEnergy \(2015\)](#).

Carmona et al., 2012) which differ in their specification of the underlying cumulative emissions process which determines the payoff of the allowance at maturity. A key modeling choice is how to incorporate the feedback of price onto emissions rate, as this can be specified as an optimal control problem for a central planner (as in Seifert et al., 2008) or alternatively as an automatic abatement produced by the structure of the market and in particular the merit order for electricity (as in Carmona et al., 2012). The latter is an example of the class of structural models for commodity prices (see, for example Pirrong, 2012; Carmona and Coulon, 2012), which avoid full agent-based equilibriums in favor of prescribing a realistic but tractable transformation from supply and demand factors to prices based on dominant characteristics of market structure.

Research contribution

For the New Jersey SREC market, we propose a structural model for price formation which parallels the equilibrium price formation for emissions allowances described in the literature above. In particular, in SREC markets as in cap-and-trade, regulated companies face a compliance deadline each year at which time sufficient credits must be submitted to avoid paying a penalty (per unit short), in this case known as the solar alternative compliance payment (SACP), and typically chosen to decline in future years as solar becomes more competitive. Compared to cap-and-trade, the uncertainty in the market thus shifts from demand for allowances (driven by an emissions process) to the supply of certificates (driven by a solar generation process), with the regulator now fixing demand (requirement) instead of supply (cap). Furthermore, the emissions abatement caused by high carbon prices has a natural parallel with the interdependence between SREC prices and installation of new solar capacity. We exploit these important similarities in the construction of our SMART-SREC price model, but also face various new challenges specific to RECs, as banking, borrowing and withdrawal rules differ, and generation responds to price quite differently, with a prominent time lag.

Therefore, our first contribution to the literature is to propose and implement an original stochastic price model for SRECs, to our knowledge the first of its kind for these young and rapidly-growing environmental markets. We analyze its key features and sensitivities, introduce parameter estimation techniques and provide a dynamic programming solution algorithm for the price as a function of the state variables. Next, we turn our attention to model calibration and backtesting, showing that our methodology is both intuitive and successful in reproducing historical price behavior in New Jersey, despite an evolving regulatory landscape and frequent rule changes. We note that even for the much-studied carbon markets, despite a number of recent empirical studies of EU ETS allowance prices (cf. Daskalakis et al., 2009; Paoletta and Taschini, 2008; Uhrig-Homburg and Wagner, 2009), we are unaware of any that fit to observed data via a structural or equilibrium model. Unlike the challenges of obtaining reliable high-frequency emissions data across many European countries (and complicated by carbon offset supply), SREC issuance is a single publicly available monthly time series (see NJCleanEnergy, 2015) making the market an excellent candidate for testing this type of model. Following the validation of our model via NJ data, our final key contribution is the analysis of various properties of the resulting price dynamics and their dependence both on regulatory parameters (like requirement level, banking and SACP rules) and on market parameters (like the level of the feedback from price onto solar generation growth rates). We use simulation analysis to show that uncertainty in both price and quantity is highly time, state and scenario dependent, and we explore how regulatory tools can significantly alter price behavior, topics which should be of significant interest to policy makers, electric utilities and solar investors alike.

The New Jersey SREC market

In recent years, many states in the US have introduced a renewable portfolio standard (RPS) to stimulate the growth of solar and other renewable energy sources that are typically not yet competitive with other traditional fuels on a cost basis. As an integral tool for RPS compliance, many states have started REC markets, and/or SREC markets in the case of a specific solar energy target (a 'solar carve out'). Among the now 30 states with enforceable RPS standards, 14 states have set up markets to trade SRECs. Bird et al. (2011) provide a comprehensive summary of their development up through summer 2011. Among these, the New Jersey market is by far the most dominant, with the highest recorded prices so far at nearly \$700 (per MWh) and the most ambitious future requirement levels at 4.10% of the state's electricity usage by 2028. While SREC markets in the US are all relatively small and young, they are projected to grow rapidly in the near future, from around 520 MW in 2011 to around 7.3 GW in 2025 (Bird et al., 2011), with about half of that total coming from New Jersey. In the year 2013 in New Jersey, 1.3 million SRECs were issued (i.e., from 1.3 GW h of power), which at recent prices of about \$150 corresponds to about \$200 million of revenues for solar generators.

New Jersey switched in 2007 from an earlier rebate program that incentivized RPS compliance to a market-based SREC program in 2007, giving us about seven years of SREC price history to study, as provided by Flett Exchange (FlettExchange, 2015) and shown in Fig. 1a. Fig. 1b shows the same period's historical annualized SREC issuance rate, corresponding to 12 times the observed monthly issuance. This allows for easier visual comparison with the annual requirements also plotted, although the relationship is masked by the strong seasonality in issuance caused by weather and daylight hours. Certificates are issued with 'vintage years', corresponding to the 'energy year' (EY) in which the electricity was produced, where EY2008 (say) refers to the 12-month period ending on May 31, 2008. As can be seen in the price history, different vintage SRECs can trade at the same time and at different prices, although they are typically highly correlated. Price differences stem from the banking rules, which describe for how many years in the future each SREC may be used for compliance. Currently SRECs

have five-year lifetimes, meaning that 2012 SRECs may still be traded until summer 2016. The exact lifetime of an SREC is slightly complicated by the so-called ‘true-up period’, a period of several months (currently six, from June 1 to November 30) between the end of the energy year and the compliance date when load-serving entities (LSEs) must submit their SRECs or pay the penalty (the SACP). It is important to note that unlike many carbon emission markets, there is limited banking but no borrowing from the next year in SREC markets, since future supply is a random variable anyway. Furthermore, paying a penalty this year does not imply a debt to produce an additional SREC the following year (known as ‘withdrawal’ in emission markets), which means from a pricing perspective that the SACP set upper bounds on SREC prices.

The challenge of understanding the NJ SREC price history is complicated by the fact that the rules have changed several times, including banking rules (SREC life), the SACP values, requirement values and the true-up period length. Although not exhaustive, Table 1 summarizes the main rule changes needed to understand price history, and which we shall use for the backtesting of our model. Since the market’s formation, SREC life has been extended from one to three to five years, the SACP has been increased significantly but then reduced again, and the requirement for 2014 onwards was recently more than doubled.³

The motivation for the most significant of these changes can be well understood in conjunction with the historical data of Fig. 1. The early years were all slightly undersupplied, leading to SREC prices very near the historical SACP values (and arguably also leading to the 2008 legislation to increase the SACP and stimulate growth). In light of the rapid exponential growth in SREC issuance in 2011–2012 (relative to a more linear requirement schedule), it is easy to understand why the rule change of 2012 was needed, in an attempt to correct an extreme oversupply. Prices nonetheless declined very rapidly from over \$600 in mid-2011 to under \$100 in late 2012, which in turn has recently led to a noticeable slowdown in the generation growth rate in Fig. 1b for EY2014. While Fig. 1 gives us a good overview, the relationship between price and generation growth rates is complicated by the time lag present in the market’s response to price movements. Fig. 2 gives us a clearer picture, with the annual growth of solar installations (line plot) plotted alongside the average SREC price of the previous calendar year (bar plot).⁴ The striking co-movement emphasizes the importance of including a time lag of approximately one year when capturing this feedback effect.

The SMART-SREC price model

We now introduce our stochastic model of the NJ SREC market in continuous time, with time indexed by t and measured in years, starting June 1, 2000 for convenience. Thus time $t=7$ corresponds to the end of energy year 2007, i.e. May 31, 2007. We also index energy years by $y \in \mathbb{N}$; year $y=7$, for example, corresponds to the time interval (6,7).

A stochastic model for SREC prices

SREC market prices, like prices in any other market, depend on the market demand (determined by regulations), and supply (i.e. SREC generation). In a single period model (one year and no banking), the value of an SREC at maturity (compliance date) must equal either the SACP if total generation is below the requirement, or zero if the requirement is met. This is clear by no arbitrage and forms the starting point for the analogous equilibrium pricing result to that of Carmona et al. (2010) in the carbon market setting; namely that the price at an earlier time must be a discounted expectation of this final payoff under the appropriate risk-neutral pricing measure. As SRECs are traded assets just like CO2 allowances (with no storage and delivery costs or constraints like physical commodities), this martingale condition must also hold here to ensure no arbitrage in the market. For simplicity, we shall assume a risk-neutral world, and focus instead on the key dependencies in SREC markets which would still be valid with the addition of typical assumptions on risk premia.

The rate of SREC generation at time t is a random variable, and is denoted by g_t (MWh/year). The single-year framework described above leads to an SREC price (for energy year y) of p_t^y at time $t \in [y-1, y]$ given by

$$p_t^y = e^{-r(y-t)} P_t^y \mathbb{E}_t \left[\mathbf{1}_{\left\{ \int_{y-1}^y g_u du < R_t^y \right\}} \right],$$

where r is a constant interest rate, R_t^y and P_t^y denote SREC requirement and SACP respectively for energy year y respectively, as observed at time t ,⁵ and $\mathbb{E}_t[\cdot]$ represents a conditional expectation given the information set at time t . Note that at time t , $\int_{y-1}^t g_u du$ is known, so the expectation can be written as the probability $\mathbb{P}\left\{ \int_t^y g_u du < C \right\}$ for a known constant C .

Usually SRECs are valid for τ additional years after their vintage year (currently $\tau=4$ in NJ), requiring an extension of the formula above. We let b_t represent the accumulated number of SRECs banked from previous years⁶ plus the new supply this

³ In addition, the requirement schedule moved from a percentage-based system originally (e.g., 0.16% of total electricity for EY2009) to an absolute system (e.g., 306 GW h in EY2011), then back to a percentage based system (e.g., 2.05% in EY2014). Whenever requirements are set in percentage terms, we use projected numbers in MWh from Flett Exchange (FlettExchange, 2015). Recent projections from mid 2014 show a slight drop relative to Table 1 due purely to changes in expectations of total power demand.

⁴ Note that EY 2014 numbers for generation have been estimated using data through March 2014.

⁵ Note that we require t dependence on R and P only to capture historical rule changes (e.g., for EY 2014, $R_{11}^{14} = 772$ and $P_{11}^{14} = 625$, while $R_{12}^{14} = 1708$ GW h and $P_{12}^{14} = 339$ following the major 2012 legislation), however we do not explicitly model requirement levels as random variables.

⁶ Note that for simplicity we do not specifically track how much of each vintage year makes up this pool of banked certificates. As long as LSEs behave rationally by submitting older SRECs first (i.e. as in a FIFO inventory rule), then there is very little chance of SRECs expiring worthless, particularly given

Table 1
Regulatory parameters in NJ, including major historical rule changes.

Energy year	True-up (mos.)	Oldest rules (no banking)		2008 change (to a 3-year life)		2012 change (to a 5-year life)	
		R	P	R	P	R	P
2007	3	32,743	300				
2008	3	65,384	300				
2009	4	130,266	300	130,266	711		
2010	4	195,000	300	195,000	693		
2011	6			306,000	675		
2012	6			442,000	658	442,000	658
2013	6			596,000	641	596,000	641
2014	6			772,000	625	1,707,931	339
2015	6			965,000	609	2,071,803	331
2016	6			1,150,000	594	2,360,376	323
2017	6					2,613,580	315
2018	6					2,829,636	308
2019	6					2,952,857	300
2020	6					3,079,139	293

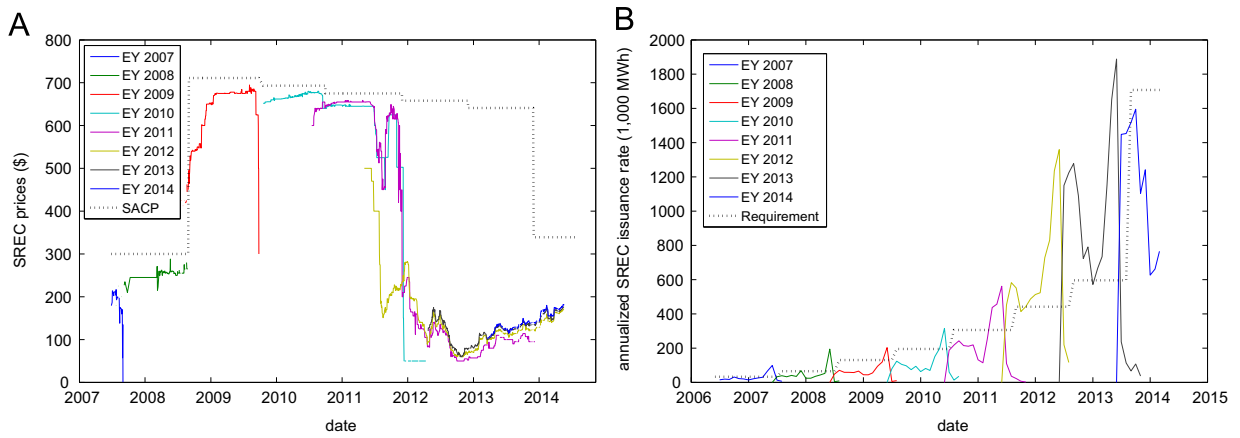


Fig. 1. Historical NJ SREC prices and generation for EY 2007–14 (note that when banking is allowed, multiple vintages trade simultaneously). (a) Daily average prices. (b) Monthly SREC issuance rate.

year, defined as

$$b_t = \begin{cases} \max\left(0, b_{t-1} + \int_{t-1}^t g_u du - R_t^t\right) & t \in \mathbb{N}, \\ b_{\lceil t \rceil - 1} + \int_{\lceil t \rceil - 1}^t g_u du & t \notin \mathbb{N}. \end{cases} \tag{1}$$

Note that exactly at a compliance date ($t \in \mathbb{N}$),⁷ b_t is taken to mean the remaining supply immediately following compliance, and therefore equals zero whenever the requirement is not met. Hence, the market price at time t for SRECs generated in energy year $y(y \leq \lceil t \rceil)$ can be obtained by

$$p_t^y = \max_{v \in \{\lceil t \rceil, \lceil t \rceil + 1, \dots, y + \tau\}} e^{-r(v-t)} P_t^v [1_{\{b_v = 0\}}]. \tag{2}$$

This formulation is similar to the multi-period carbon price formulation discussed by Hitzemann and Uhrig-Homburg (2011), but with the important differences that firstly SRECs cannot be banked for an indefinite number of years, and secondly ‘withdrawal’ is not required. As in the carbon setting, the martingale condition on prices does not necessarily hold exactly at a compliance date, since at this time a cashflow might be generated by selling an SREC to a power supplier for the

(footnote continued)

five-year lifetimes and growing requirements. It is however straightforward to extend this formulation to a vector valued b_t as in Khazaei et al. (2014), where the possibility of SRECs expiring each year is specifically tracked.

⁷ For simplicity we ignore the ‘true-up’ period mentioned in the section ‘The New Jersey SREC market’, which effectively shifts the compliance date backwards by several months, but does not meaningfully change the methodology or results. For the backtesting in the section ‘Model validation: historical price comparison’, we adjust our implementation to handle this feature, but ignore it in other sections.

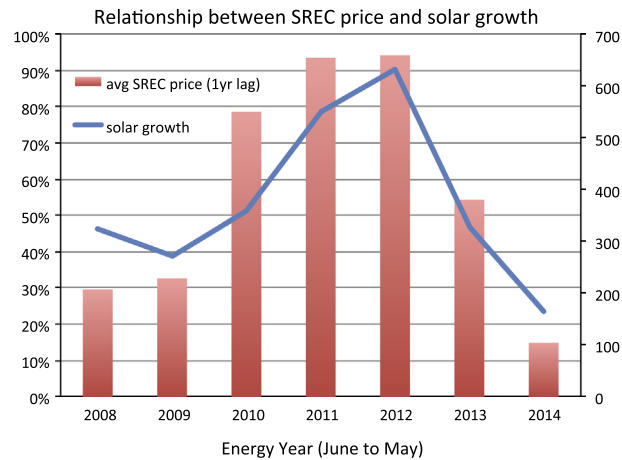


Fig. 2. Annual growth rate of new solar installations (line plot, left axis) and average SREC price in the previous year (bar plot, right axis).

SACP and then buying another one back for much less just after compliance. Such a price drop at $t \in \mathbb{N}$ represents the loss of one of the $\tau + 1$ opportunities to use the SREC for compliance,⁸ which are captured by the maximum function in (2).

A stochastic model for SREC generation

To compute market prices from (2), we need to know the density function for g_t (in SREC/y) for all future compliance years. Motivated by observed SREC issuance patterns, we model g_t in most general form by

$$\ln(g_t) = a_0 + a_1 \sin(4\pi t) + a_2 \cos(4\pi t) + a_3 \sin(2\pi t) + a_4 \cos(2\pi t) + a_5 t + a_6 \int_0^t \bar{p}_u du + \varepsilon_t, \quad (3)$$

where the ‘feedback price’ $\bar{p}_t = f(p_u; 0 \leq u \leq t)$ represents some increasing function of historical prices, reflecting the intuitive behavior (if $a_6 > 0$) that industry installs more capacity when prices (typically current or recent) are higher. In addition, SREC generation varies with daylight hours and weather conditions; e.g. generation in summer is expected to be more than in winter. These seasonal patterns⁹ are modeled with the sine and cosine functions in (3). Finally, uncertain changes in generation (i.e. noise) are modeled by a mean-zero stochastic process ε_t , which we shall assume to be stationary and independent at each of our time steps, but could also be made mean-reverting.

Note that (3) can be rewritten as

$$g_t = \hat{g}_t(p) \exp(a_1 \sin(4\pi t) + a_2 \cos(4\pi t) + a_3 \sin(2\pi t) + a_4 \cos(2\pi t) + \varepsilon_t), \quad (4)$$

where having deseasonalized and removed noise, $\hat{g}_t(p)$ now represents average annualized rate of SREC issuance, which is proportional to the total installed capacity of solar (in MW) at time t . As mentioned earlier and witnessed in recent data from New Jersey, new investment in solar generation is directly dependent on SREC market prices. The generation model above captures this effect, since it implies that the state variable \hat{g}_t grows at a price-dependent rate:

$$\frac{d}{dt}(\ln(\hat{g}_t)) = a_5 + a_6 \bar{p}_t \quad \text{for } a_5 \in \mathbb{R}, a_6 > 0 \quad (5)$$

In the simplest case $\bar{p}_t = f(p_u; 0 \leq u \leq t) = p_t$, and all feedback from prices onto generation is immediate. While in practice there may be some instant response due to some generators choosing strategically to supply more SRECs to the market when prices are highest, the majority of long-term feedback is likely to be lagged due to new project construction time which can take many months or even years. In addition, different investors may look at different historical SREC price averages in order to make their investment decisions. We shall discuss various possibilities for \bar{p}_t in the next section as we fit the New Jersey data.

The parameter a_6 here represents the sensitivity of generation growth to the average market price, while a_5 captures the rate of solar growth independent of the incentives provided by an SREC market. Finally, we note that a linear form is

⁸ If generators are behaving optimally, then the SACP is only paid when all existing SRECs have been used up for compliance, implying that this price drop should not occur for a particular vintage (as it disappears from the market), but rather reflects the drop when comparing the old vintage to the new one.

⁹ Even though prices are not themselves seasonal and compliance depends on the sum of SREC generation over a full year, it is still vital to include seasonality in our generation model in order for the resulting relationship between p_t^y and accumulated supply b_t to make sense between compliance dates (i.e., given a current value b_t , we need to account for which seasons have already passed, and which remain, in order to accurately determine the probability of reaching the requirement at the next compliance date).

assumed in (5) for simplicity and to capture key features, a choice which will be justified by our data fitting exercise in the next section. Many other modeling choices are of course possible. For example, one might postulate a threshold level of SREC prices at which point most new investors would join. Alternatively, one might attempt to capture investor risk aversion by incorporating both price levels and price volatility into the feedback structure. However, given our risk-neutral framework and our desire to focus on the dominant first order effects, we do not include such extensions here.

Solution algorithm

In order to solve for the SREC price p_t^y (i.e. at time t , for vintage year y) it is important to notice firstly that the right hand side of (2) contains a dependence on the price p_t^y (via \bar{p}_t^y) which cannot be conveniently rearranged. Our price model is a function of generation (i.e. probabilities of ending below or above the SREC requirement in future years), while our generation model is a function of price (i.e. the market responds to prices when investing in new solar). This feedback is a central feature of our model and is a natural consequence of an equilibrium model for prices, but as a result the numerical solution algorithm requires some care. Nonetheless, we can use dynamic programming to solve for p_t^y backwards through time on a discretized grid, as a function of its state variables, notably the accumulated SREC total b_t and the issuance rate \hat{g}_t . If $\bar{p}_t^y = p_t^y$ for simplicity (immediate feedback), then our solution surface at each t depends only on these two variables, while in all other cases a third dimension must be added at the cost of computation time. The fundamental idea in the solution algorithm is that discounted SREC prices satisfy the martingale at all time points except at compliance dates, where a maximum function appears in the equation, similar to the exercise opportunities for an American option. Further details are included in the appendix, along with some parameter sensitivity analysis.

Parameter estimation

To fit our model to the NJ SREC market, we estimate all parameters by maximum likelihood estimation (MLE), or equivalently by linear regression, using historical data from the period July 2008 to March 2014. We remove the earliest two years of the data which provide much less value for estimating current behavior, as solar penetration was very low (and SREC prices flat with low trading volumes). As we observe historical generation data at a monthly frequency, we discretize our model with time steps of $\Delta t = 1/12$ (one month) throughout. Hence (with t_0 equal to July 1, 2008)

$$\ln(g_t) = a_0 + a_1 \sin(4\pi t) + a_2 \cos(4\pi t) + a_3 \sin(2\pi t) + a_4 \cos(2\pi t) + a_5 t + a_6 \sum_{u=t_0}^{t-\Delta t} \bar{p}_u \Delta t + \varepsilon_t. \tag{6}$$

Our primary goal is to estimate parameters a_0, a_1, \dots, a_6 and find an appropriate distribution function for ε_t . However, at the same time, we must identify a suitable ‘feedback price’ \bar{p}_t^y , specifying a functional form for f . We consider and test three alternatives on the data:

- *Lagged Historical Price*: We simply set

$$\bar{p}_t^y = p_{t-\gamma}^y$$

where $\gamma \in \{0, 1/12, 2/12, \dots\}$ is the lag parameter measured in years. Fig. 2 from earlier suggests we might expect a best fit of γ to be close to 1.

- *Exponentially Weighted Moving Average (EWMA)*: We update \bar{p}_t^y each month via

$$\bar{p}_t^y = \delta p_t^y + (1 - \delta) \bar{p}_{t-\Delta t}^y \quad (\text{where } \bar{p}_{t_0}^y = p_{t_0}^y),$$

where $\delta \in (0, 1]$ is a parameter which weights history less as it increases.

- *Lagged EWMA (combination of the two above)*: We might argue for a lag due to construction time as well as an averaging over history due to the behavior of investors making decisions, in which case we can update \bar{p}_t^y as follows:

$$\bar{p}_t^y = \delta p_{t-\gamma}^y + (1 - \delta) \bar{p}_{t-\Delta t}^y \quad (\text{where } \bar{p}_{t_0}^y = p_{t_0-\gamma}^y), \tag{7}$$

Since the third formulation above (lagged EWMA) includes the previous two as special cases (for $\gamma = 0$ and $\delta = 1$ respectively), we can effectively compare all three versions at once by varying γ and δ . Fig. 3 uses a heat map to compare the sum of squared errors (SSE) obtained for each pair (γ, δ) , and reveals that the best fits occur either for γ near 10 months or for shorter lags if instead δ is made very small. Clearly the two parameters γ and δ have related effects. For example $\delta = 0.1$ stretches the exponentially weighted average so much that the mean of the weights can be calculated to be at about 9 months in the past, making an additional lag unnecessary.

Overall there is very little difference between the calculated SSE values on the main red arc running from lower left to middle right in the heat map, suggesting that any choice in this region would be reasonable. Given the evidence from Fig. 2, the intuition that there should indeed be a construction time lag, and the added simplicity of using only one ‘lag mechanism’ instead of two, we choose to set $\delta = 1, \gamma = 10/12$ (i.e., the optimal γ parameter for the simple lag case with no EWMA). The

parameter estimates for (6) along with 95% confidence intervals are provided in Table 2, including σ , the standard deviation of the noise ε_t . Only the parameter a_5 is found to not be significantly different from a null hypothesis of zero at the 5% level. The remainder of the parameters is given in Table 3, with the interest rate fixed at $r = 2\%$ throughout for simplicity, as it has very little impact on results.

Fig. 4a shows the model's ability to capture well both the linear trend and seasonal components of $\ln(\hat{g}_t)$ observed in the NJ data. Notice the slight leveling off of the trend in recent months (as discussed earlier) and the ability of the model to replicate this via the price feedback mechanism. Fig. 4b shows a Q–Q plot of the distribution of noise ε_t , and suggests that it

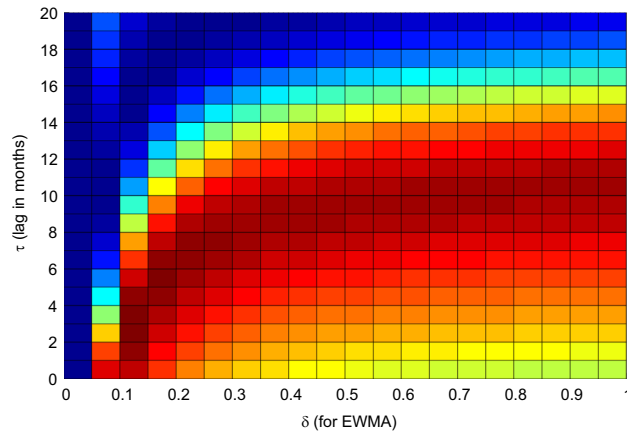


Fig. 3. Heat map of SSE for different pairs (γ, δ) , which determine feedback time lag via (7). Note that red indicates the lowest SSE (best fit). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

Table 2
Estimated generation parameters $\{a_0, \dots, a_6\}$ and σ .

Parameter	a_0	a_1	a_2	a_3	a_4	a_5	a_6	σ
Estimated value	10.9558	-0.1209	0.0900	0.2151	0.3859	-0.0151	1.27×10^{-3}	0.1863
Confidence intervals (95%)	10.5385	-0.1875	0.0235	0.1486	0.3184	-0.1923	0.94×10^{-3}	0.1509
	11.3730	-0.0543	0.1565	0.2816	0.4535	0.1621	1.60×10^{-3}	0.2217

Table 3
Additional parameter values.

Parameter	γ	δ	r
Value	10/12	1	0.02

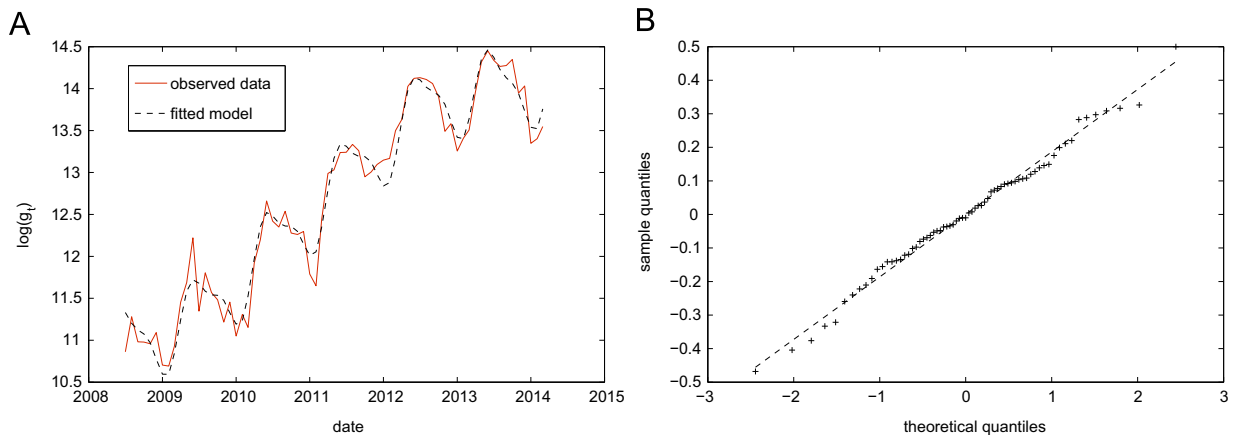


Fig. 4. Data fitting results for historical SREC generation g_t , using (6). (a) Fitted generation function. (b) Q–Q plot for the remaining noise.

can be quite well characterized by a normal distribution. This is confirmed by a Kolmogorov–Smirnov test of normality, in which the null hypothesis is not rejected at the 1% significance level. We estimate the noise to have a normal distribution with mean zero and variance 0.035 (i.e. $\varepsilon_t \sim N[0, 0.035]$).

While we are encouraged by the ability of the model to fit the data, we note also that the early years of this market may no longer be very representative of the situation today, since it grew rapidly from a very low base, averaging a growth rate of over 60% per year. The a_5 and a_6 parameter together imply that for a range of prices $[0, 700]$, the range of possible generation growth rates is $[-0.015, 0.876]$, while for the more recent caps of around \$300 on prices, growth can be as much as 0.367. However, as the market becomes more saturated, or solar starts having other substantial effects on the grid and on power markets, we can expect this growth rate to slow. Since US solar markets are at a fairly early stage of penetration (relative for example to some European countries like Germany), we argue that it is still reasonable (and advantageous) to study the SREC market in isolation, instead of building a more complicated joint model for both electricity and certificate price dynamics. The 2028 requirement of 4.1% solar is still many years away and we do not expect significant interdependencies or correlations with power markets to manifest themselves before then.¹⁰ Similarly, the model could also potentially be extended to additional sources of risk or randomness such as determining and including realistic probabilities of future rule changes, but we leave such additional challenges for future work.

Model validation: historical price comparison

One key criterion for assessing the model's performance is its ability to mimic observed historical price movements. Recall that, as in the spirit of traditional equilibrium or structural price models, data on fundamental price drivers (i.e. SREC issuance rates, market rules, etc.) formed the basis of our parameter estimation procedure, not prices themselves (with the exception of lagged prices used indirectly in the generation model, not directly to fit a price process). Hence, we should not expect to replicate a high level of detail in price dynamics, but instead for overall patterns to be consistent. In particular, with only monthly SREC generation data available, we clearly cannot predict features like daily price volatility or reaction to specific market news and announcements. Nonetheless, the structure of the model allows us to capture well key long-term effects in the markets like rule changes and significant shifts in the rate of SREC issuance.

Fig. 5 shows the results of the historical comparison for all SREC vintage years studied, with observed prices in the first plot and model prices in the other three (for different values of γ). The former are simply monthly averages of the data in Fig. 1 while the latter are produced by stepping through the saved price surfaces $p(t, b_t, \hat{g}_t)$ and inputting the observed values of b_t : (cumulative generation adjusted by subtracting the requirements as described by (1)) and \hat{g}_t (annualized issuance approximated by using the last 12 months of observations). In addition, we switch between different saved price surfaces when the rules changed,¹¹ as summarized in Table 1 earlier. We test out three different values of the lag γ (reestimating $\{a_0, a_1, \dots, a_6\}$ in each case) to illustrate the role of this parameter in accurately replicating price drops and rallies. The plots are generally encouraging, as they show that the model captures well the primary characteristics of historical price dynamics, such as the rapid price jump in 2008 associated with the first rule change, the high prices during 2009–2011, and the drop to low levels more recently. The timing of overall movements is generally well matched, as are some subtler details: e.g., the slight dip in prices in 2011 (driven by higher than expected summer and fall issuance) before a return towards the penalty at compliance, and the low prices for the EY2012 certificates in this same period (due to the fact that they were not valid for 2011 compliance – no ‘borrowing’ allowed).

Price behavior in the early years of the New Jersey market was clearly dominated by rule changes and a general undersupply of SRECs, with prices hugging the penalty throughout. Therefore, the much more interesting and relevant period of study is 2011–2014, when prices suddenly dropped in 2011 as growth of solar overtook the requirement schedule, then declined further in 2012 before reversing this trend in 2013 and 2014. Fig. 5 reveals that the model predicts the general price decline and rise correctly, but getting the exact timing and magnitude correct depends somewhat on the estimated γ . For example, $\gamma=0$ produces immediate price feedback that slows the drop of prices much more abruptly as generation growth rates slowed in tandem, while our estimated $\gamma = 10/12$ fails to slow the drop quite fast enough as it fails to ‘see’ the historical price collapse until late 2012. However, ultimately all model plots show recoveries to similar price levels in 2014.

Table 5 shows the average absolute value of percentage errors for each energy year in Fig. 5. Overall, the most accurate prediction by this measure comes from our estimated value of $\gamma = 10/12$, although visually the $\gamma = 5/12$ case in the middle of the three is perhaps most convincing regarding the slowing of the recent price drop described above. Although average

¹⁰ One might argue that even when SREC markets are too small to affect power markets, solar generation growth rates may be affected by power price movements since generators receive income from selling electricity as well as selling certificates. Also, SREC requirement levels (when fixed in percentage terms) depend on overall power demand. However, both mean levels and typical changes in average annual power prices (in the PJM power market) are currently significantly lower than the corresponding values for SRECs. Demand is also relatively stable from year to year, implying that neither of these power market drivers is likely to explain much price variation currently. Nonetheless, this could certainly change in the future and is an interesting area for further research, but is quite challenging to assess from available historical data.

¹¹ More specifically, due to the two rule changes (2008 and 2012) and the 8 vintages under consideration (2007–2014), we require a total of 13 different saved price surfaces. We also note that while the newest rules were not officially enacted until June 2012, the proposals outlining the likely changes were publicly available in late 2011 (e.g., see history of blogs on [SREctrade, 2015](#)), and hence likely to be ‘priced in’ already. We therefore use January 2012 as the date for the second rule change, with September 2008 appropriate for the first. We could alternatively use a weighted average of prices under different regulatory regimes to reflect changing market expectations of upcoming rule changes, but such assumptions are likely to be rather ad hoc.

errors of more than 30% may sound very high, it is important to note that these are not the results of directly fitting a price model, but rather these are predictions from a low-dimensional structural model which looks only at its few state variables to determine a price at each time. Moreover, we see that a large portion of the error is concentrated around 2012, when dramatic price changes and low prices easily lead to large errors if the exact timing of the price swings is not replicated. While details of price movements are very challenging to replicate without adding extra complexity to the model, year-on-

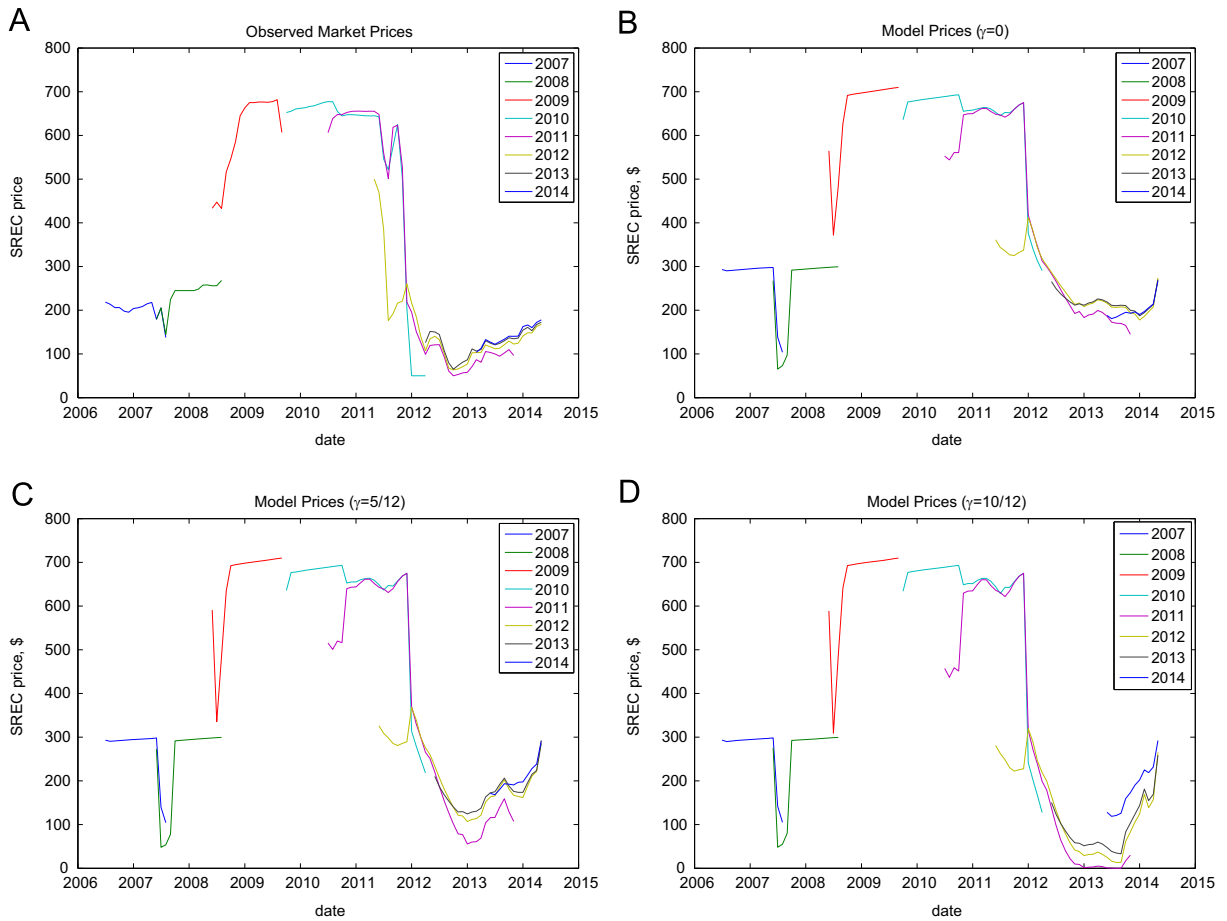


Fig. 5. Historical comparison of market versus model prices for all years in the study (and choices of $\gamma \in \{0, 5/12, 10/12\}$). (a) Market prices (monthly averages). (b) Model prices ($\gamma = 0$). (c) Model prices ($\gamma = 5/12$). (d) Model prices ($\gamma = 10/12$).

Table 4
Average absolute percentage error (vs. data) by energy year for each case plotted in Fig. 5.

Year	2007	2008	2009	2010	2011	2012	2013	2014	Avg.
$\gamma=0$	41.5	29.2	12.2	4.2	3.8	133.0	158.7	47.9	53.8
$\gamma=5/12$	41.4	30.9	13.0	4.2	4.9	105.8	52.1	38.2	36.3
$\gamma=10/12$	41.4	30.9	13.3	4.3	6.7	71.9	44.3	40.2	31.6

Table 5
Percentage changes in annual average prices by energy year (note: final column is the average of absolute values of other columns).

Year	2007–2008	2008–2009	2009–2010	2010–2011	2011–2012	2012–2013	2013–2014	Avg.
Data	9.7	144.8	18.4	–2.1	–52.4	–69.5	47.3	49.2
$\gamma=0$	–17.0	151.7	11.8	–5.6	–28.5	–51.5	–10.5	39.5
$\gamma=5/12$	–18.0	155.0	11.6	–6.8	–32.4	–68.7	41.8	47.8
$\gamma=10/12$	–17.8	154.0	11.8	–8.6	–37.2	–85.2	110.4	60.7

year changes in average price levels are a criterion much better captured by the model, as illustrated in Table 4. Here we see that recent prices changes are particularly well captured by the $\gamma = 5/12$ lag, with 2012–2013 producing a 68.7% decrease (versus 69.5% in reality) and 2013–2014 a 41.8% increase (versus 47.3% in reality).

The lag in feedback, γ , is a particularly tricky parameter to estimate and could plausibly change over time. As the market matures, many of its participants may indeed develop a stronger understanding of price behavior and consequently an ability to anticipate future shocks or rule changes, thus shortening the time lags imposed by physical constraints of construction. Moreover, it is worth remembering that while the requirement schedule is fixed throughout 2012–2014 in our analysis, a percentage-based requirement schedule implies that participants are constantly re-evaluating their expectations of future requirements (given expected total future energy demand). Several other complications could arguably have weakened results, such as the time lag between actual solar power generation and SREC issuance. The profile of this delay appears to have changed somewhat as the market has matured, with huge spikes in issuance in June of the early years in Fig. 1 becoming much less prominent in later years.¹² Many such minor factors not included in the model could potentially explain price differences, such as information on planned solar installations, unobservable market expectations (of both future issuance levels and of rule changes), forecasts of power demand, and even information like changes to solar technological development and cost. Nonetheless, and despite the complexity of the market under consideration, we argue that the overall picture is very reasonable and thus strongly justifies our modeling approach.

Model application: scenario analysis and policy considerations

As we have seen, SREC markets are particularly prone to regulatory changes, typically intended to restore a reliable price signal following a supply demand imbalance. While regulators may have the flexibility to respond to such events, frequent rule changes are clearly undesirable. We now deploy the SMART-SREC model together with Monte Carlo simulation to analyze some of the regulatory decisions faced by policy makers of SREC markets, the existing tools at their disposal, and the consequences for price behavior. Similar to the backtesting of the previous section, for each scenario we first generate and save a series of price surfaces for each vintage and each parameter set under consideration, before stepping forward through these saved surfaces within the simulations.

Requirement schedule

As can be understood from (2), requirements play a very important part in determining market prices. As shown in Fig. 1, market prices fell dramatically in the beginning of 2012. This motivated the new regulation of 2012 that increased the yearly requirement greatly for EY2014, followed by a decelerating rate of requirement growth afterwards (specifically, 21.3%, 13.9%, 10.7%, 8.3% in years 2015–2018, then roughly 4% thereafter, as in Table 1), producing a concave future requirement schedule. On the other hand, we saw in the section “Parameter estimation” that generation tends to grow exponentially even though its growth rate can slow as prices fall. In Fig. 6, we compare projected future market prices from the current regulation with an alternative policy imposing exponential growth on the yearly requirement. We use the alternative requirement function $\hat{R}_t^y = R_t^{14} \exp(\beta(y - 14))$ for some constant β , which imposes an exponential scheme from $y = 14$ (EY2014) onwards. We do not match our initial conditions to the market today, but instead choose our values (for June 1, 2013) to be $b_{13} = 0$ and $\hat{g}_{13} = \hat{R}^{14} \exp(-\beta/2)$, such that around the middle of the first year, the expected annualized SREC issuance rate matches the requirement. We compare two different levels of feedback parameter a_6 , and in both cases choose $\beta = a_5 + 150a_6$ such that our alternative requirement growth rate lies in the middle of the range of feasible growth rates i.e. matching the generation growth rate for prices at \$150).

The two plots show that an exponential scheme for yearly requirements can produce long-term stable prices with low variability provided that the growth rate is chosen appropriately by regulators to account for the level of feedback of prices onto investment behavior. Under the current regulations, the results tend to be more volatile overall, and prices drop towards zero over the coming years. A high level of feedback onto new supply (Fig. 6b) slows generation growth more effectively and prevents the prices from collapsing completing to zero, but prices nevertheless remain very low in later years. Of course the alternative requirement case is artificially stable in the sense that parameters have been chosen specifically to keep supply and demand in balance as much as possible, and the feedback mechanism in the model further reinforces this equilibrium. We also recognize that the long-term generation growth rate and corresponding parameter values may drop significantly as solar penetration becomes high (requiring model recalibration), but we nonetheless point out that our calibrated currently model predicts that the existing NJ requirement schedule does not grow fast enough to keep prices from again collapsing towards zero in the coming years. Only time will tell if and when further legislative intervention will be required.

¹² As solar generators are not required to register their SRECs immediately after actual generation, some may choose to wait until just before the compliance deadline (in the true-up period) to register a large quantity at once. Others may simply prefer using an annual metering system to a monthly one. Finally, all SREC issuance is subject to a delay of up to a month simply due to the metering, reporting and processing time (see PJM's tracking system GATS (PJMEIS, 2015) for more on these details).

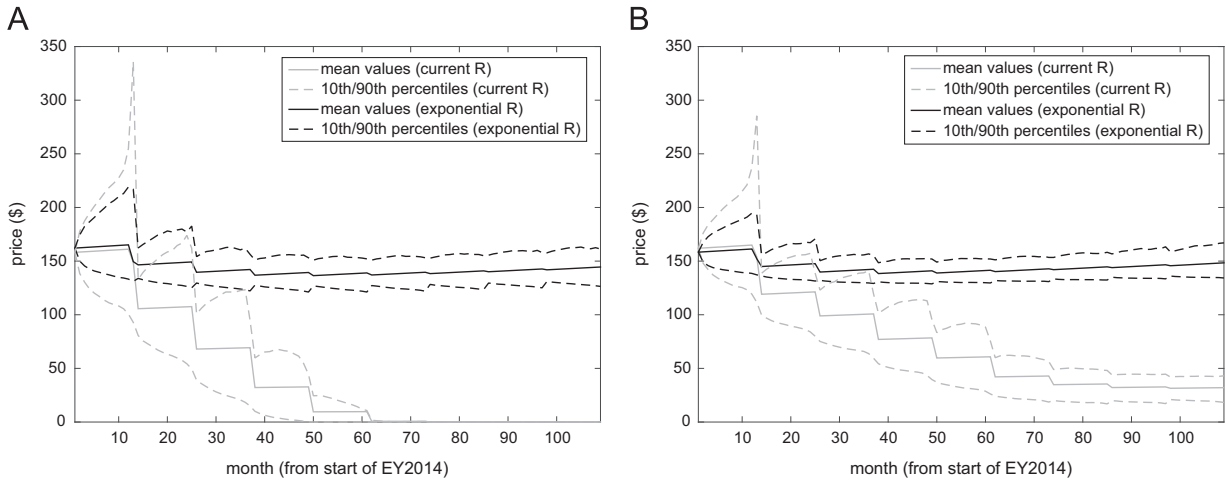


Fig. 6. Comparison of SREC prices (newest vintage) under the two requirement schedules (R^y vs. \bar{R}^y), for two different choices of a_6 : mean prices (solid lines) and 10th/90th percentiles (dashed lines) from 10,000 simulations. (a) Low feedback ($a_6 = 6 \times 10^{-4}$). (b) High feedback ($a_6 = 12 \times 10^{-4}$).

Feedback and banking

Given our observations above, for now we fix the requirement schedule to be \bar{R}^y instead of R^y , as it provides long term SREC prices centered conveniently between zero and the SACP, thus more practical for scenario analysis. We begin by investigating the implications of different levels of feedback a_6 in the market, as well as the role played by the regulator's banking parameter τ . It is well known that the introduction of banking in certificate markets will reduce price volatility, but we also examine its impact on the quantity of SRECs generated and banked and the resulting profit distribution for investors in solar.

Specifically, for each scenario of parameters, we run 10,000 ten-year simulations starting in mid 2013 (EY2014), with b_{13} and \hat{g}_{13} fixed appropriately as before. In Fig. 7, we plot the means and standard deviations (as a function a_6) of each of the following random variables:

- *Cumulative SRECs generated*: total solar energy produced, $\int_{13}^{23} \frac{1}{12} g_t dt$.
- *Solar capacity installed by year 10*: annualized generation rate at the final time, \hat{g}_{23} .
- *SRECs banked in year 10*: final accumulated supply of unused certificates, b_{23} .
- *Year 10 prices*: the price (of the newest vintage) at the final time, p_{23} .
- *Average prices (years 1–10)*: average of the newest vintage price at each t , $\int_{13}^{23} \frac{1}{10} p_t^{[t]} dt$.
- *Cumulative revenues (market)*: total revenues for the entire market, $\int_{13}^{23} \frac{1}{12} g_t p_t^{[t]} dt$.

Note that we do not focus on the values that these random variables take, and instead normalize all six of them by their starting values (at $a_6 = 4 \times 10^{-4}$) in order to plot everything on the same axes. Several interesting observations can be made. Firstly, as expected and also shown in Fig. 10a, greater feedback in the market lowers price volatility, and by as much as 70% when comparing $a_6 = 20 \times 10^{-4}$ to $a_6 = 4 \times 10^{-4}$. On the other hand, price means are relatively flat in the experiments, since we adjust a_5 along with a_6 to keep the range of generation growth rates equally centered throughout. Looking at quantity instead of price, standard deviations of cumulative SREC generation also decrease with a_6 , because feedback in the market acts to respond to the noise in the generation process to keep long term supply of SRECs in balance. For example, early years of low solar generation lead to higher SREC generation in later years due to the response in the generation growth rate. However, perhaps counter-intuitively, the standard deviation of the *installed* solar by year 10 increases as feedback increases, because the range of possible generation growth rates given by (5) is wider. In other words, the feedback effect responds to prices which reflect the history of generation (through b_t), not just the current generation rate \hat{g}_t . Hence more volatile trajectories of g_t can occur in the simulations, even if the integral over each trajectory tends to be more stable. This interesting finding is our first example of a price versus quantity tradeoff, with an increase in price stability accompanied by a decrease on the quantity side.

Clearly a_6 is not a tunable regulatory parameter (except perhaps through educating market participants), but rather a market characteristic of which all participants should be aware. From an investor perspective, the fifth random variable plotted above (average prices) can be interpreted as the average annual revenues per SREC of a solar generator with a constant production over time. Therefore, a risk-averse investor may only be willing to pursue new solar projects if feedback levels are observed to be high enough (meaning other future investors might back out if prices decline), lowering price risk and corresponding revenue uncertainty. From a policy making perspective, comparing Fig. 7d to Fig. 7b illustrates how the introduction of two banking opportunities ($\tau = 2$) can help to control the additional uncertainty in installed solar quantities

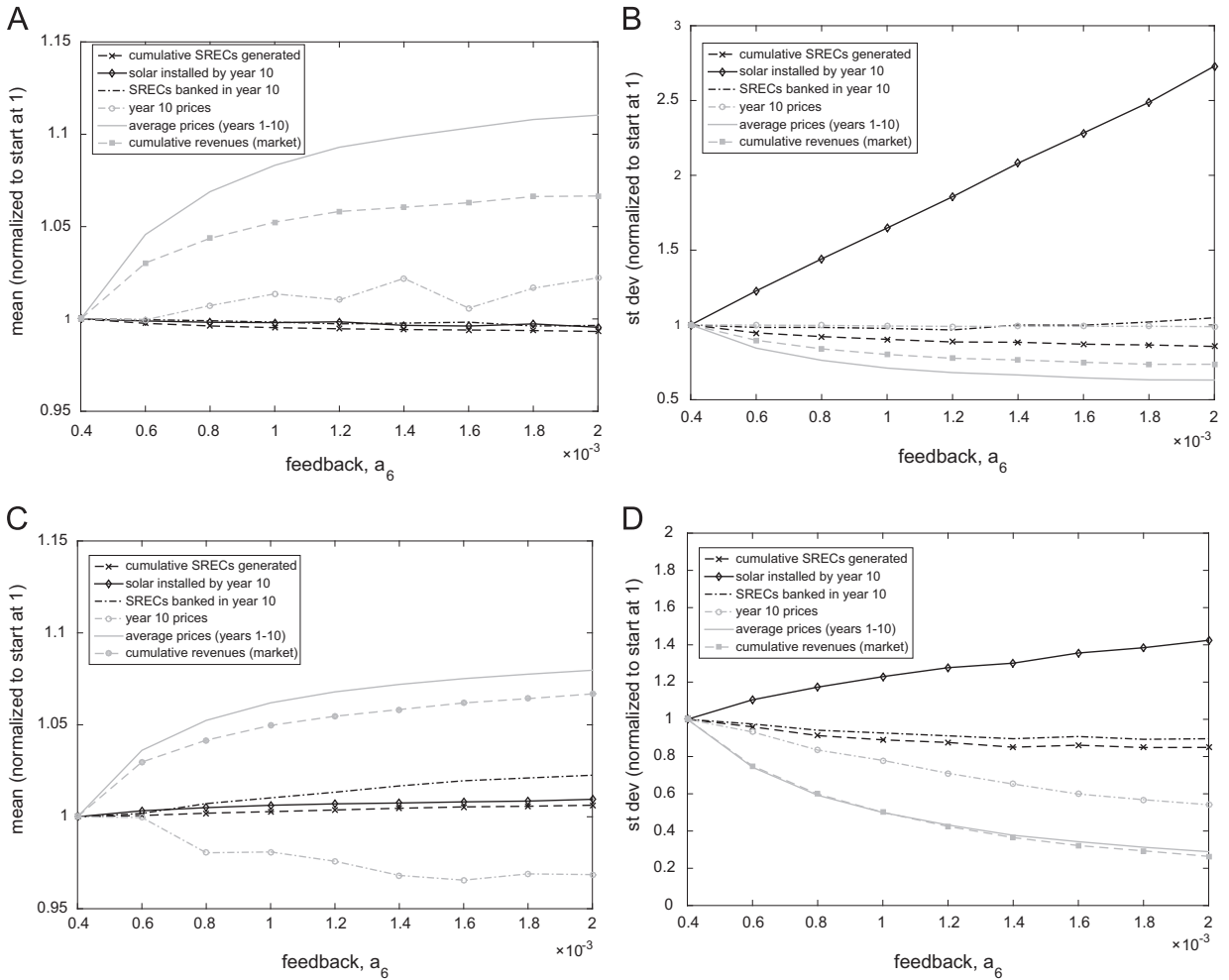


Fig. 7. Simulation results for six chosen random variables (see text): means and standard deviations vs. a_6 for two different banking rules. (a) Means ($\tau = 0$). (b) Standard deviations ($\tau = 0$). (c) Means ($\tau = 2$). (d) Standard deviations ($\tau = 2$).

for high a_6 . Price volatility is also further dampened (potentially encouraging new investment), but as we shall explore in the next section, increased banking can also come at a cost in the long-term.

Alternative design for SACP

The New Jersey SREC market has recently experienced an ongoing period of substantial oversupply, characterized by the historically low prices witnessed in Fig. 1. While the five year life of certificates has clearly helped to prevent prices from dropping even further (as they are supported by the chance of a return to undersupply in a few years' time), banking possibilities inevitably also prolong periods of oversupply, since certificates then rarely ever expire. In this subsection, we consider an alternative to banking, which we show can still help prevent price collapses while also avoiding such scenarios of large accumulated oversupply in which old vintages repeatedly fulfill compliance needs and thus hamper long term growth. As regulatory goals may differ by region or even by time period (for example, as markets mature), we do not attempt to formulate a precise regulatory objective, but simply consider two simple and intuitive criteria that most regulators should support:

- Criterion 1: Meeting long-term targets for solar sector growth (i.e., $\hat{g}_y > R^y$).
- Criterion 2: Avoiding price collapses to zero (i.e. $p_y^y > 0$).

Due to the natural tradeoff between price and quantity certainty, these two criteria cannot both be guaranteed, but a regulator can aim to adjust market design to maximize the probability of both. Price modeling and simulation can help to achieve such goals.

Much of the intrinsic instability (and the corresponding need for frequent rule changes) in all markets for tradable certificates – both RECs and carbon allowances – can be traced back to the ‘cliff-like nature of the SACP or penalty function. Just below the requirement, the penalty is several hundred dollars per MWh, while just above, it is suddenly zero. A natural question to explore is whether something smoother could be implemented, with a penalty which moves gradually from zero to P^y in a region around R^y . In Khazaei et al. (2014), a ‘sloped’ SACP function with these characteristics is proposed and analyzed, along with other adaptive policy ideas to enhance market stability. In particular, the idea of the ‘slope’ replacing the ‘cliff’ consists of replacing the indicator function inside (2) with a newly defined alternative SACP function $f_t^{SACP}(x)$

$$f_t^{SACP}(x_t) = \begin{cases} P_t, & x_t < (1-\lambda)R_t \\ P_t - \frac{P_t}{2\lambda R_t} (x_t - (1-\lambda)R_t), & (1-\lambda)R_t \leq x_t < (1+\lambda)R_t \\ 0, & x_t \geq (1+\lambda)R_t \end{cases} \quad (8)$$

where $\lambda > 0$ is a parameter chosen by regulators to control the steepness of the slope (or the width of the corresponding ‘requirement region’ $[(1-\lambda)R_t, (1+\lambda)R_t]$) and x_t is the quantity of SRECs submitted for compliance at time t . Although this change is straightforward to understand and implement, when introduced in combination with banking possibilities, it adds much greater complexity to the price formation mechanism.¹³

Here, we focus solely on the simpler case of using f_t^{SACP} with no banking ($\tau = 0$), which avoids extra complexity and instead stresses the role of the slope purely as an alternative to banking. Furthermore, we investigate the impact of f_t^{SACP} under three different scenarios, varying between existing and hypothetical requirement schedules and initial conditions:

- *Scenario A*: Exponentially growing \tilde{R}^y with ‘balanced’ $b_{13} = 0, \hat{g}_{13} = 1.56$ m.
- *Scenario B*: Exponentially growing \tilde{R}^y with oversupply $b_{13} = 1$ m, $\hat{g}_{13} = 1.25$ m.
- *Scenario C*: Existing R^y (Table 1) with oversupply $b_{13} = 1$ m, $\hat{g}_{13} = 1.25$ m.

The initial conditions on b_{13} and \hat{g}_{13} for Scenarios B and C are roughly in line with the situation in New Jersey at the end of EY2013. Interestingly, while b_{13} set to 1 million implies a large oversupply of certificates, installed solar \hat{g}_{13} is lower than in Scenario A due to a jump in the requirement under the recent 2012 rule change. These combine to put the market in a challenging situation to meet targets on \hat{g}_y , and emphasises the crucial difference between criteria $b_y > R^y$ and $\hat{g}_y > R^y$, since annual compliance can be met without the need for any new capacity. In Fig. 8, we investigate the probability of meeting the two criteria on a year-by-year basis, and consider two different sloped penalties (varying λ , with $\tau = 0$) along with two different banking rules (varying τ , with $\lambda \rightarrow 0$), as well as the base case of no slope and no banking.

Under Scenario A (the assumptions of the previous subsection), the base case clearly performs worst both in terms of avoiding price collapses and in growing installed solar. Adding either banking possibilities or a slope increases the probability of meeting targets, with higher values of λ and τ tending to do even better. Interestingly, when it comes to prices hitting zero, we see that even a modest slope ($\lambda = 0.1$) almost eliminates this risk, while banking of course does the same. In contrast, in the base case, prices hit zero a little more than 50% of the time, as they must converge to either zero or P^y at each compliance date.

Scenario B instead considers the large oversupply of 1 million banked certificates at the start of EY2014. We now see a disadvantage of allowing banking, as the oversupply and corresponding low (but not zero) prices stifle growth of new solar for several years before the market can catch up. (Note that four banking chances in this case at first performs a bit better than two due to the feedback effect since prices stay a bit higher.) In practice such times are likely to lead to pressure for regulatory fixes through rule changes. Of course in the sloped SACP cases (no banking allowed), prices are zero at first but the (admittedly unrealistic) initial oversupply is immediately wiped out after year 1. Finally, in Scenario C we combine a large oversupply with a slowly growing requirement schedule, as is the reality in New Jersey. We see even weaker performance in terms of the first criterion, especially in the banking cases. The chance of meeting targets in the early years is zero, then the market catches up a little in the middle years, before facing continued oversupply due to flatter requirement schedules in the long term. In contrast, after a slow start, the sloped SACP approach without banking fares much better, easily achieving the targets on installed solar in the later years. However, this comes at the expense of an increasingly high risk of prices collapsing to zero in later years, due to the slow requirement growth.

The results above again illustrate the natural tradeoffs between controlling prices and controlling quantities. Clearly, encouraging too much banking makes it less likely that targets for solar will be reached if periods of oversupply and stagnating prices develop. On the other hand, banking strongly mitigates the risk of prices collapsing to zero completely, destroying credibility in the market mechanism as a whole. A regulator might also choose to combine these two policy tools in order to try to get the best of both. Many scenarios can be compared in such cases, but the resulting market behavior then

¹³ The crucial difference between this setup and the simpler ‘cliff’ one is that x_t need not simply equal $\min(R_t, b_t)$, as there is a non-trivial decision required of market participants as to how many SRECs to bank, with a tradeoff between today’s (choice of) penalty and expected future prices. For detailed analysis of the optimal banking decisions in this framework we refer the interested reader to Khazaei et al. (2014) where several fairly technical theorems are presented and proved regarding this proposed market design.

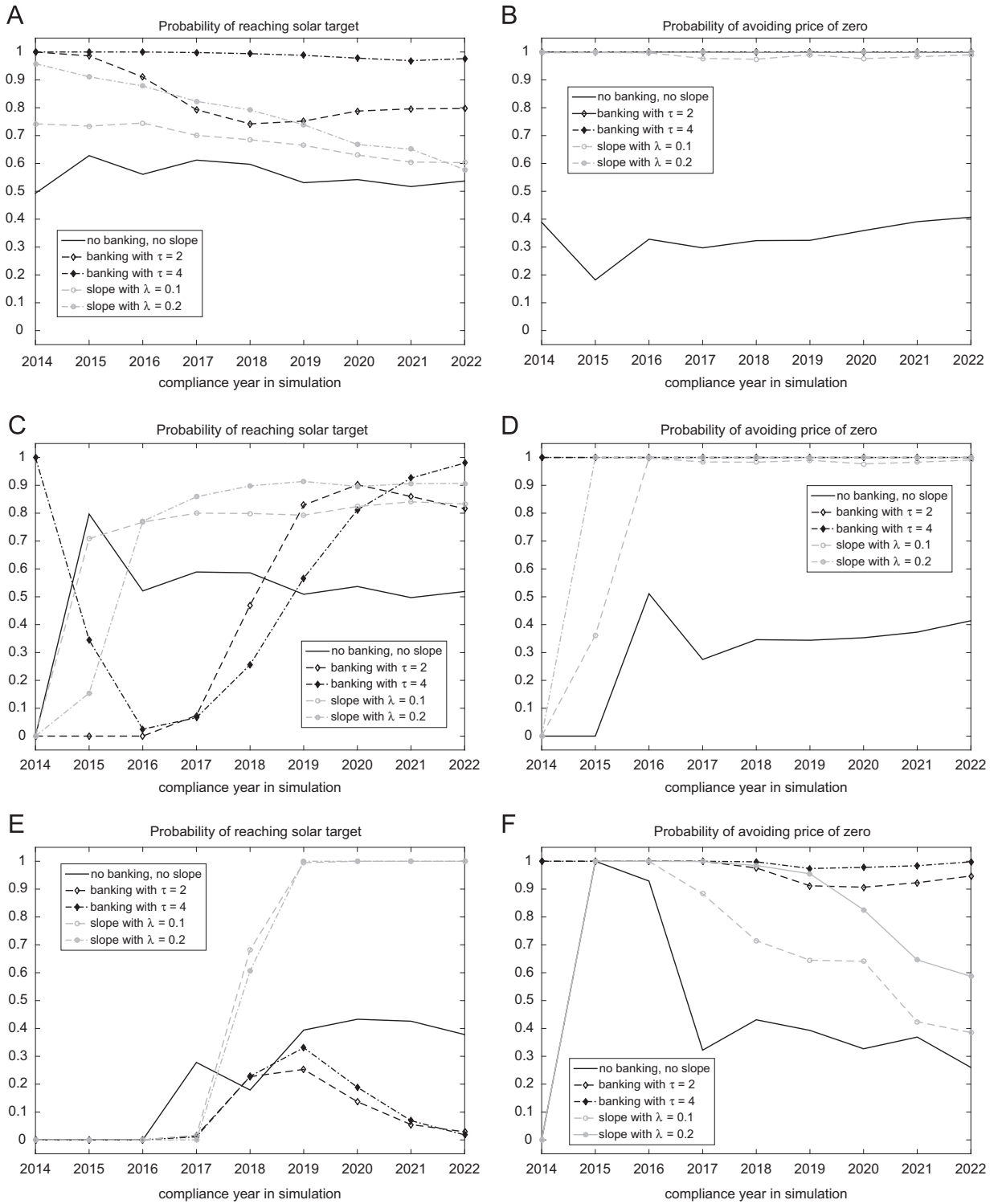


Fig. 8. Simulation results when varying τ and λ : probabilities of reaching solar targets and avoiding price collapses for three different scenarios. (a) Criterion 1, Scenario A. (b) Criterion 2, Scenario A. (c) Criterion 1, Scenario B. (d) Criterion 2, Scenario B. (e) Criterion 1, Scenario C. (f) Criterion 2, Scenario C.

depends also on the optimal certificate submission decisions of participants, relying on the more advanced theory developed in [Khazaei et al. \(2014\)](#). Nonetheless, even in the simpler cases presented here, we clearly see the strong need for a tractable and empirically supported structural model for SREC prices, our primary contribution of this paper.

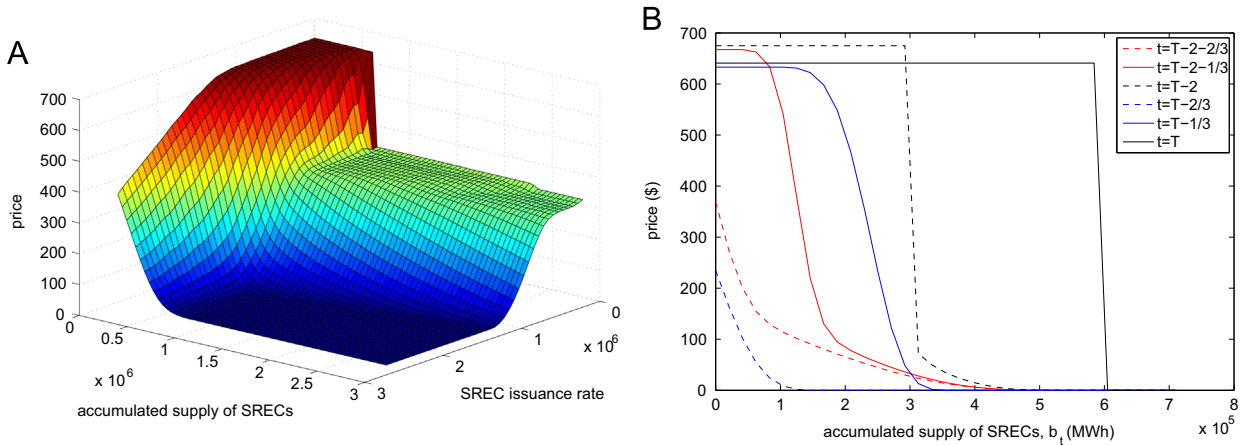


Fig. 9. A sample SREC price surface (2013 vintage at $t=12.5$) and sample cross-sectional price curves (2011 vintage, for various t). (a) Price surface $p_t^{13}(b_t, \hat{g}_t)$ at $t=12.5$. (b) Price curves $p_t^{11}(b_t)$ for different t .

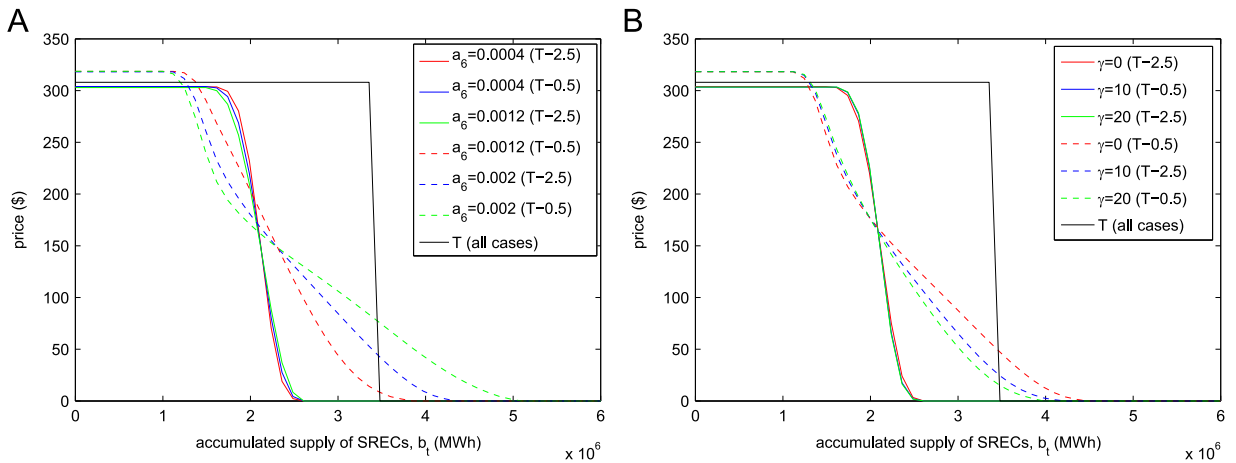


Fig. 10. 2010 SREC price curves $p^{10}(b_t)$ for various t , and several a_6 or γ . (a) Price sensitivity to a_6 . (b) Price sensitivity to γ .

Conclusion

The coming years may prove to be critical ones for the future of the New Jersey SREC market, as policy makers debate how to dampen volatile price dynamics and to effectively encourage the growth of the solar sector. As evident from our analysis, SREC prices can be highly sensitive firstly to market design, including banking rules, choice of requirement growth rate and penalty mechanism, and secondly to the behavior of market participants such as the response of new generation to price movements. Given the structural similarities to cap-and-trade markets, it is not surprising that many of the same important research questions apply, both from a pure price modeling perspective and from that of optimal regulatory policy. For example, the carbon market literature has featured various studies of optimal market design, including the comparisons in Gruell and Taschini (2011) of various alternative proposals like price collars and allowance reserves, and proposals of dynamic allowance allocation schemes in Carmona et al. (2010). In the market for renewable energy credits, such issues are especially relevant research topics today, as other states and regions look to New Jersey as a guide for future REC markets of their own.

In this work, we have proposed a new model, SMART-SREC, that can begin to answer such questions by understanding the key factors and relationships driving SREC prices, contributing to their volatility and their sometimes surprising price swings. In particular, we have successfully calibrated to and back-tested against the early years of the New Jersey market, obtaining reasonable parameter estimates and replicating historical price movements with the model. We have also demonstrated the important role played by market design in determining price behavior, discussed one possible alternative design feature, and analyzed economic implications and tradeoffs via simulation analysis. Such insights illustrate a crucial advantage of structural price models that exploit fundamental supply and demand relationships and market rules. Historical SREC price behavior would be extremely challenging to capture or understand in a reduced-form or classical econometric price model. In order to attempt to build plausible future SREC price distributions for managing risk or making investment

decisions, we have seen that it is particularly critical to accurately model the underlying regulatory structure of the markets. We hope that our work can pave the way for a growing literature in this fascinating field of environmental economics.

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Appendix A. Solving for the price surface

Here we describe in detail our methodology for solving (2) numerically for the SREC price p_t^y (i.e. at time t , for vintage year y) as a function of the state variables. In the full SMART-SREC model proposed in the section "The SMART-SREC price model", the accumulated SREC total b_t , the issuance rate \hat{g}_t and the average historical price \bar{p}_t^y are all state variables, implying a three-dimensional array is needed for p_t^y at each time step. In fact, to be more precise, three dimensions are needed for $\gamma = 0, \delta < 1$, but a $2 + \gamma/\Delta t$ dimensional array is needed for $\gamma > 0$ unless an approximation technique is employed instead.¹⁴ However, by setting $\tau = 0$ and $\delta = 1$ in (7), we can reduce the dimensionality of the model to 2, since p_t^y replaces \bar{p}_t^y in (5). Although somewhat less realistic in practice, this simpler case of *immediate* feedback preserves the main qualitative features of the model and is hence very useful for analysis given its faster computation time.

The central observation needed to implement a numerical solution of (2) is that discounted SREC prices satisfy the martingale condition at all time points except at compliance dates. Nonetheless, the dynamic programming algorithm requires some care, primarily because the right hand side of (2) cannot be simplified to remove dependence on p_t^y , the value which we need to solve for at each point in our discretized space. This is of course due to the feedback of price on future generation rates, which affects the calculation of each of our expectations, and is a natural consequence of an equilibrium model for prices.

In the more general case ($\delta < 1$) the algorithm proceeds as follows:

- We first discretize space by choosing a grid of values for b_t, \hat{g}_t and \bar{p}_t . Time is discretized in monthly steps throughout to match the frequency of historical generation data. For b_t and \hat{g}_t we choose a different grid for each SREC vintage year y due to the growth of solar, with lower bounds zero throughout, and upper bounds sufficiently above $R^{y+\tau}$, the highest relevant requirement. Similarly for \bar{p}_t , we choose an upper bound of $\max\{P^y, \dots, P^{y+\tau}\}$, the highest relevant penalty.¹⁵ Finally, we discretize the distribution of the noise ε_t using spacing appropriate to the grid for b_t .¹⁶
- We next initialize the dynamic program by evaluating the payoff of the SREC at the end of its life, i.e. at $t = y + \tau$. At this time, all is known so no expectation appears in (2), and the price $p_{y+\tau}^y \in \{0, P^{y+\tau}\}$.
- We work backwards through time, solving the martingale condition at each grid point: $p_t^y = \exp(-r\Delta t) \mathbb{E}_t[p_{t+\Delta t}^y]$. Since the right hand side is bounded, continuous and decreasing in p_t^y by model construction,¹⁷ a unique solution p_t^y always exists. The system of Eqs. (1)–(3) forms a fixed-point problem that can be solved iteratively via various standard root finding algorithms. (We use Matlab's 'fzero' function.)
- At times $t = y + \tau - i$ (for $i = 0, \dots, \tau$) in the backwards dynamic program, we must incorporate the possibility that the SREC price may jump up to the SACP value if the requirement has been missed for the compliance year just ended. In other words, the martingale condition may not hold at a compliance time, as can be seen in (2). This is implemented by taking the maximum of the discounted expected future value and the immediate value (the penalty times indicator of compliance today).¹⁸

¹⁴ The exponentially weighted moving average (EWMA) approach conveniently reflects all of history with a single extra dimension, due to the 'memoryless' property of the exponential function. In the case of a true lag (with $\gamma > 0$), we instead need to store an entire vector of length $\gamma/\Delta t$ of historical prices, which clearly very soon becomes computationally intractable for dynamic programming. As a simple approximation, instead we include as a state variable only $\bar{p}_{t-\gamma\Delta t}$, and when we require a value for $\bar{p}_{t-\gamma\Delta t+\Delta t}$ in our algorithm, we choose $\bar{p}_{t-\gamma\Delta t+\Delta t} = \frac{1}{\gamma}\bar{p}_t + \frac{\gamma-1}{\gamma}\bar{p}_{t-\gamma\Delta t}$ as our best estimate, i.e. we linearly interpolate today's price with the γ -lagged price instead of using the $(\gamma-1)$ -lagged price. The impact of this approximation is very small.

¹⁵ This is typically simply P^y since SACP is normally set by the regulator to decrease over time (as is currently the case), while the requirement always increases. The boundary conditions are straightforward to implement given the boundedness of the payoff function in (2).

¹⁶ We note that in any case linear interpolation is required in the algorithm since (3) produces values for \hat{g}_t and b_t which do not fall exactly on grid points when calculating the expectation described below.

¹⁷ More specifically, this can be proven via the price bounds discussed in the section "A stochastic model for SREC prices" and the fact that b_t is continuous and increasing in g_t (from (1)), while g_t is continuous and increasing in \hat{g}_t (from (4)), and the drift of \hat{g}_t is continuous and increasing in p_t^y (from (7) and (5)).

¹⁸ This can be thought of as an optimal exercise decision ('value if we decide to bank for the future' versus 'exercise value for this year's compliance'), but is in fact automatic in the sense that no model is needed to make the right decision (as we know whether the requirement has been reached). Hence, a better analogy from finance might be a stock or bond that pays a dividend or coupon only if some observable event occurs.

We comment that the computation time for this exact dynamic programming algorithm can become quite large for very fine grids in three space dimensions (e.g. if $\gamma > 0, \delta = 1$). If we choose approximately 50 grid points for each and a five year SREC life (60 time steps), this requires a few hours to solve in Matlab. If any more dimensions were to be incorporated into the model (for example, random processes for R_t or P_t , or auto-correlated noise ε_t), approximate dynamic programming approaches could certainly be explored. However, our aim is to suggest a sophisticated but tractable structural model which identifies the price's dependence on the dominant risk factors for the SREC market. Hence, we often choose $\gamma = 0, \delta = 1$ to allow for much finer space grids with shorter computation time.

Appendix B. Analysis of solution surface

Fig. 9 illustrates some typical results from the SREC price model, solved via our dynamic programming algorithm described in detail above. The surface plot shows the price of a 2013 vintage SREC at a fixed time $t = 12.5$ (December 2012), six months before its first compliance date for this SREC (ignoring the true-up period for simplicity). As expected, SREC prices $p_t^y(b_t, \hat{g}_t)$ are always decreasing in both accumulated supply b_t and generation rate \hat{g}_t , since an increase in either state variable means a higher chance of the market meeting the requirement and not needing the SREC for compliance. We also see clearly the convergence of the surface to the 2013 SACP for low values of b_t and \hat{g}_t , and in this case a striking drop down to the new 2014 SACP value for slightly higher values of b_t .

Price surfaces emerge through a diffusing and smoothing (backwards in time) of the certificates' terminal values and intermediary compliance values. Further insight can therefore be gained by looking at cross-sectional plots like Fig. 9b, which shows the price of a 2011 vintage SREC versus b_t at various point in its 3-year life.¹⁹ Time $t = y - 1$ is the beginning of the SREC's life, and we let T represent the final expiry of the certificate, which for 2011 SRECs corresponds to mid 2013 ($T = y + \tau$) if the true-up period is ignored, or late 2013 ($T = y + \tau + 1/2$) with six months of true-up. Note also that when we are at a compliance date, we have plotted the SREC's price curve immediately *before* compliance. Hence we clearly see the discontinuity at the requirement level, due to the indicator function multiplied by the penalty amount (SACP). We also observe the gradual flattening and diffusion of the curves to the left as we move backwards in time, with jumps to the right and upwards on compliance dates. Finally, it is interesting to note that at the first compliance date of the SREC's life (at $t = T - 2$), the price is equal to the SACP if $b_t < R_t^c$, but is not immediately zero for $b_t > R_t^c$. This right tail of the price curve implies that in this case there is some value to banking surplus SRECs for future periods (as long as it is not an extreme surplus).

Of course these price curves and surfaces are sensitive to the level of price feedback in the market, as well as the time lag in its occurrence. While both these values can be estimated from history, it is useful to understand how price behavior might change if these parameters changed over time or from market to market. In Fig. 10a, we fix $\gamma = 0$ and choose values of a_6 corresponding to low, medium and high feedbacks, where the middle value of 1.2×10^{-4} is close to our estimate from historical data. As feedback increases, price curves flatten, implying lower price volatility and also that uncertainty about whether the requirement will be reached tends to last longer into the SREC's life.²⁰ Furthermore, the sensitivity to a_6 is logically more significant earlier in the certificate's life (i.e., at $t = T - 2.5$ in the plot) when the feedback has more time to have an effect.

Similarly, in Fig. 10b, we investigate the impact of the time lag γ in the feedback effect by letting γ vary between 20 months (very long lag), 10 months (medium lag) and 0 (immediate feedback). The results show that delaying the feedback steepens the price curves as functions of b_t , effectively weakening the overall feedback effect. However the impact is relatively small throughout (even for several years before maturity and for a rather extreme $\gamma = 20/12$) and qualitative features of results are preserved. Although not shown here, very similar results are found when varying the EWMA parameter δ , with a decrease to δ acting like an increase toy, but again having only a minor effect. This helps to justify our preference in the section "Model application: scenario analysis and policy considerations" to use the $\gamma = 0, \delta = 1$ case (immediate feedback) for scenario analyses, due to the computational benefit of fewer dimensions. Nonetheless, the strength and timing of the feedback in the market is in practice an important consideration for all market participants.

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¹⁹ For a fair comparison, cross-sectional plots choose \hat{g}_t values at each time appropriately to reflect the typical growth of generation over time that corresponds to average price levels in the model. Similarly we fix $\bar{p}_t^y = P_t^y/2$, and all other parameters are as in Table 2.

²⁰ Note that as we change a_6 , we also adjust a_5 such that the generation growth rate when price is \$150 remains constant. Thus we effectively widen the range of possible generation growth rates in both directions (instead of just upwards) and keep prices centered at the same point (instead of always decreasing prices).

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